A

DESCRIPTION

OF THE

LINES drawn on GUNTER's SCALE,

AS IMPROVED

By Mr. JOHN ROBERTSON,

Late Librarian to the Royal Society;

AND EXECUTED

By Meſſrs. NAIRNE and BLUNT,

Mathematical Inſtrument Makers, Cornhill, London.

WITH THEIR

Uſe and Application to Practice, exemplified more
eſpecially in

NAVIGATION AND ASTRONOMY.

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By WILLIAM MOUNTAINE,
Mathematical Examiner to the Honourable Corporation of Trinity-House of Deptford-Strond, and F.R.S.

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BRITISH MUSEUM

To the Right Honourable and Honourable the
COMMISSIONERS of the Board
of LONGITUDE.

LORDS and GENTLEMEN,

THE rapid Progreſs which has been made within theſe few Years paſt in the Art of Navigation, is, no Doubt, principally to be attributed to the Attention and Encouragement given by Your Board to Improvements in Inſtruments and Aſtronomy; it therefore gives us the higheſt Satisfaction that we are able to acquaint the World that the Plan of the Marine Scale, of which this Publication is explanatory, has been examined and approved by You.

We are,

Lords and Gentlemen,

Your moſt obliged, and

Moſt obedient humble Servants,

NAIRNE and BLUNT.

PREFACE.

MR. John Robertſon, late Librarian to the Royal Society, formerly Maſter of the Royal Mathematical School at Chriſt's-Hoſpital, and afterwards Head-Maſter of the Royal Marine Academy at Portſmouth, was not more known in the Mathematical World by his uſeful Publications, than eſteemed for his ſtrict Attention to, and faithful Diſcharge of the ſeveral Truſts committed to his Care.

This deſerving Man, not long before his Death, was applied to by ſome Gentlemen of Science, together with Mr. Nairne, to inſpect into, and correct the Errors which had crept into † *Gunter's Scale*, by Length of Time, and Negligence in the Fabrication.

He ſet about this Work, though then in a State of Health much impaired, re-conſtructed new Lines, with Improvements, which were mechanically finiſhed under his own Inſpection, by Meſſrs. Nairne and Blunt, Mathematical Inſtrument Makers, in Cornhill.

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† Mr. *Edmund Gunter* the Inventor of this Inſtrument, was Profeſſor of Aſtronomy in *Greſham* College, and firſt publiſhed the Scales of Logarithms, Numbers, Sines and Tangents, in his Book of the Deſcription of the Croſs-Staff, in the Year 1624.

A Deſcription of the *Lines*, with the Inſtrument, was then thought neceſſary, which he began, but his Pen dropt, (he died) before a bare Account of *theſe*, in rough Copy, was compleated.

The long, intimate and uninterrupted Friendſhip which had ſubſiſted between the deceaſed and myſelf, (almoſt forty Years) induced me to inſpect them, when they were found in the unfiniſhed State aforeſaid; but a bare Deſcription of the *Lines* compleat, would not have been ſufficient to authorize a Publication, without ſome Account of their Uſe, and Application to Practice, exemplified: This Taſk then I have endeavoured to perform, from the Reſpect I bear to the Memory of my deceaſed Friend, and Regard to his ſurviving Family, with a Deſire to aſſiſt in whatever tends to Nautical Improvements.

As this Inſtrument has always been held in Eſteem for its quick Diſpatch of Buſineſs, eſpecially in MARINE PRACTICE, I have more immediately exemplified its Uſe in *that* Branch, which may alſo be ſufficient to illuſtrate its Application to any other, where a Proportion is neceſſary, and can be obtained. Hence, the Reader need not be advertiſed that he ought to be acquainted with *Principles* and *Proportions* before he can properly make Application to the Scale; and it is rationally ſuppoſed that every one who deſires to make Uſe of it, hath, or will previouſly make himſelf Maſter of theſe preparative Elements.

A Courſe, then, of Plane and Spheric Geometry, and Trigonometry with the Conſtructions at large, was not intended here, by the *original Plan*, ſince it would have encreaſed the Expence, and was alſo thought unneceſſary, as theſe Subjects are ſcientifically treated of and explained by many good Authors, whoſe Books are extant, and eaſily obtained.

Notwithſtanding this primary Intention, I have ſomewhat encroached upon the Size of the Tract, by adding the Mode of Operation with Compaſſes, as well as by the Slider, being ſufficiently ſatisfied that the firſt will be frequently wanted both at Land and Sea, but be that as it may, the Directions would appear imperfect without it; ſome Particulars are alſo enlarged upon, eſpecially the Principles of Mr. EDWARD WRIGHT's Meridional Parts, and a few others in Spherics, which it is hoped will not be unacceptable: I ſhall only add, That if this *Opuſculum*, which was originally intended as a ſmall Piece of Public Service, ſhall be found to anſwer that Purpoſe, it will be a ſufficient Reward for the Time and Pains beſtowed thereupon, by

WILLIAM MOUNTAINE.

St. John's, Southwark,
March 5th, 1778.

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DESCRIPTION AND USE OF GUNTER's SCALE IMPROVED.

THIS Engliſh Inſtrument has been for many Years in great Eſteem among the Mariners of ſeveral Nations, and particularly thoſe of Britain, on Account of the Readineſs with which Nautical Caſes were ſolved by it: But it having, like many other uſeful Things, at length been in general careleſsly fabricated, ſo as to become a cheap Article in the Buſineſs of fitting out Seamen, it was not eaſy, even among ſome of the moſt reputable Shops of Inſtrument Makers, to meet with one divided with ſufficient Accuracy and Convenience to ſatisfy a careful Artiſt, or to accommodate Beginners; for half of one Side being moſtly taken up with two Diagonal Scales, it did not give Learners a ſufficient Choice of Scales to fit to the Size of their Paper the Figure they wanted to conſtruct: And there being no other Scale of Chords than to a Radius of 2 Inches, all the Circles deſcribed were to a Diameter of 4 Inches, which on many Occaſions were larger than convenient: Beſides, a Mercator Chart conſtructed by the Meridian Line, gave a Degree of Longitude of the Length only of about $\frac{1}{8}$ of an Inch, which was much too ſmall when applied to a particular Chart, containing but few Degrees of Latitude and Longitude. Theſe Imperfections being taken Notice of above 30 Years ſince, ſome Inſtrument Makers had new Patterns made to remove them; and a better Sort of Gunters (as they are uſually called) have been in the Shops ever ſince.

Of late, ſome ingenious Mariners, and others, have wiſhed for a Scale of this Sort ſomewhat longer, and where the Meridian Line might be fitted to a Degree of Longitude ſufficiently large to conſtruct any particular Chart by; and the Uſe of Compaſſes avoided, by having a proper ſliding Scale: To accommodate the curious in theſe Matters, the Subject has been recon-

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reconſidered by the Promoter of the aforementioned improved Gunters; and he conceives that an Inſtrument with the following Scales accurately laid on them, in the Order enumerated, will fully anſwer their Deſire.

This Scale is then made 30 Inches long, 2 Inches broad, and about half an Inch thick: One Face to contain, what may be called, the natural Scales; and the other Face to have put upon it the Logarithmic Scales; and in ſome convenient Part, is to be marked to Sea Feet, each of 13,8258 Engliſh Inches; and each Foot divided into 12 equal Parts, called Sea Inches.

Of the Face of the Natural Numbers.

Half the Length of this Face is filled with Scales of equal Parts, which are denominated by the Diviſions of an Inch; viz. Scales of 10, $12\frac{1}{2}$, 15, 20, 25, 30, 35, 40, 45, 50, the Left-hand primary Diviſion of each Scale to be divided into 10 equal Parts, and alſo into 12 equal Parts.

On the other Half of this Face is put the ſeveral Lines which conſtitute what is uſually called, the Plane Scale.

Such as the Scales of Rhumbs, Chords, Sines, Secants, Tangents and half Tangents, fitted to a Radius of 2 Inches.

Alſo the Dialling Scales, viz. Inclination of Meridians, Latitudes, Chords and Hours, to a Radius of 2 Inches,

A Scale of Chords and Miles of Longitude, to a Radius of 3 Inches, other Scales of Chords and Rhumbs, to a Radius of 1 Inch.

Of the Face containing the Logarithmic Scales.

The Breadth of this Face is divided into 12 Parts, or Scales running nearly the whole Length; nine of theſe Scales may be called fixed, and the other three moveable; being on a ſliding Piece, of about $30\frac{1}{2}$ long, $\frac{1}{2}$ an Inch broad, and of the Thickneſs of the Scale.

The Order of theſe Scales, or Lines, are,

Fixed	1 A Line of Sine Rhumbs, marked S. R. 2 A Line of Tangent Rhumbs, marked T. R. 3 A Line of Verſed Sines, marked V. S. 4 A Line of Sines, marked Sin.
Sliding	5 A Line of Sines, marked Sin. 6 A Line of Numbers, marked Num. 7 A Line of Tangents, marked Tan.

8 A Line

Fixed
8 A Line of Tangents, marked Tan.
9 A Line of Numbers, marked Num.
10 A Line of Meridian Degrees, to 50, marked Mer.
11 A Line of Meridian Degrees, from 50 to about 74, marked Mer.
12 A Line of Degrees of Longitude, marked Lon.

Along this Face an Index or thin Piece of Braſs, about an Inch broad, is contrived to ſlide, which going acroſs the Edge of the Scale at right Angles thereto, will ſhew on the ſeveral Lines the Diviſions which are oppoſite to one another; although the Lines are not contiguous. The Apparatus at the Right-hand conſiſts of a Braſs Box and two Screws; the Slider paſſes freely through the Box when the perpendicular Screw is eaſed, and may be readily ſet by Hand to the Terms given; yet to be more accurate, and to keep the Slider in its true Poſition, move the perpendicular Screw, which, by a Sub-ſpring will fix the Slider in the Box, and then by the Motion of the Horizontal Screw, the greateſt Degree of Accuracy poſſible may be obtained.

If, by Choice, or for the Sake of Variety, any Perſon wiſhes to uſe Compaſſes, as on the common Gunter, the ſame may be done here, firſt bringing the Radius on the Slider exactly in a Line with the others on the Right-hand, and there to continue fixt; but it ſhould be obſerved, that the Compaſſes have very fine Points, and even then, with the greateſt Care, they are apt to indent, and otherwiſe deface the Sub-diviſions.

Some Uſes of the Scales of Natural Numbers on the back Face of the Gunter, commonly called the Plane Scale.

I. *Of the Scales of equal Parts.*

Theſe ſeveral Scales are uſeful in the Conſtruction of Geometrical Figures, whoſe Dimenſions, or the Meaſures of their Lines, are given in Numbers, which is always done in Queſtions relating to Sailing. The Figure to be conſtructed, may be fitted to almoſt any Size thought convenient, by a proper Choice of one of theſe Scales; which to prevent Miſtakes, are marked at the Left-hand End with a Number ſhewing how many of the ſmalleſt Diviſions in each Scale goes to an Inch.

Thus, on the Scale marked 10, each of the larger, or primary Diviſions is of one Inch; and the Left hand Inch or Diviſion, is ſubdivided into 10 equal Parts; in the next Scale marked 12½, where each primary Diviſion is ⅘ of an Inch, the Left-hand Diviſion is alſo ſub-divided into 10 equal Parts, and the 12½ marked at the End, ſhews that 12½ of thoſe Sub-diviſions make an Inch: In the next Scale marked 15, each of the larger Diviſions is ⅔ of an Inch, and the Left-hand one being ſubdivided into 10 equal Parts, 5 of theſe Sub-diviſions make ⅓ of an Inch, and conſequently 15 ſuch Diviſions make an Inch; and the Number 15 is therefore marked againſt the Scale, and ſo of the reſt: When either of theſe Scales are uſed, it ſhould be particularly noted, thus, if the Scale marked 40 at the End, is choſen to take the Lengths of the Lines from, the Figure is ſaid to be done by the 40 Scale; and all the Lines relating to ſuch Figure muſt be meaſured on the 40 Scale. And the like is to be underſtood of any other of theſe Scales of equal Parts: Thus by naming the Scales uſed, Miſtakes may be prevented of applying different Scales, the Lines belonging to one and the ſame Figure.

The apparent Advantage of Diagonal Scales, may incline ſome to think theſe ſimply divided ones are not equally uſeful; but Perſons who have been ſometime in the Practice of uſing Scales, know that Lengths of Lines, even expreſſed by a Number of three Places, (ſuch as for Example 575) may be taken ſo accurately from ſimply divided Scales, as to make no diſcernable Difference between the like Numbers taken from a Diagonal Scale.

The Left-hand Diviſion of each of theſe Scales is alſo divided into 12 Parts, for the Convenience of taking Dimenſions of Feet and Inches; beſides, ſuch Diviſions of 12, furniſh the Means of taking of ſome Parts accurately; as ⅓, ¼, ⅔, ¾, &c.

EXAMPLE.

Required to take from the 30 Scale, a Line repreſenting a Length of 57 Yards.

SOLUTION.

In the Scale marked 30, ſet one Foot of the Compaſſes on that primary Diviſion marked 5, and extend the other Point to the 7th Sub-diviſion in the Left-hand Space, where it is divided into 10 Parts, and this opening of the Compaſſes will repreſent the Diſtance ſought. If the Diſtance was 575, ſet one Point on the 5th primary Diviſion, and extend the other to the Middle between 7 and 8, among the Sub-diviſions of the 10; or to the 9th Sub-diviſion of the 12; and this opening of the Compaſſes will ſhew a Diſtance equal to 575.

It muſt be obſerved, that the Point of the Compaſſes ſhould reſt on a Part of the primary Diviſion, which is in an even Line with the Sub-diviſions where the leſſer Parts are to be taken.

The Uſe of the Lines of Chords, Natural Sines, Tangents, Semi-Tangents, and Secants, is ſo well known in the Conſtruction of Mathematical Figures, that any Explanation would be unneceſſary.

The Lines of Latitudes, Inclination of Meridians and Hours, are peculiar to Dialling; on which there are ſeveral Treatiſes extant, and may be better conſulted upon Occaſion.

Of the Logarithmic Lines on the Gunter's Scale.

The firſt and moſt uſeful of all the Log. lines on the Gunter's Scale is that called the Line of Numbers, it being applicable to almoſt every Computation wherein Numbers are concerned, and many Treatiſes have been wrote to ſhew its Uſe on various Subjects; but here it is intended only to ſhew its Application ſo far as it may relate to Marine Affairs.

Of the Parts, and Notation on the Line of Numbers.

The whole Length of this Line is divided into two equal Portions or Lengths, uſually called Radius's, the firſt Radius begins at the Left-hand End with the Number 1; the middle Point between the two Radius's is alſo marked with the Number 1, and the End of the ſecond Radius on the Right-hand End, is marked with the Number 10, againſt which ſtand the Letters Num.

Each of theſe equal Radius's or Portions is divided into nine unequal Spaces, marked 2, 3, 4, 5, 6, 7, 8, 9, but the Spaces in one Radius are exactly of the ſame Lengths with the Spaces alike marked in the other Radius; each of theſe nine unequal Spaces are ſub-divided; firſt into ten unequal, and then each of theſe into two or more Parts, ſuch as thoſe Spaces can conveniently contain.

If the Left-hand 1 be called the Number one, the middle 1 ſtands for the Number 10, and the 10 at the Right-hand End ſtands for the Number 100; then the Figures 2, 3, 4, &c. in the firſt Radius, ſtand for the Numbers two, three, four, five, ſix, ſeven, eight, nine; and the like Figures in the ſecond Radius ſtand for the Numbers 20, 30, 40, 50, 60, 70, 80, and 90; and the ten firſt Sub-diviſions in the Spaces of the ſecond Radius, ſtand for the reſpective Units between the marked Spaces.

If the Left-hand 1 be called 10, the middle 1 then ſtands for 100, and the 10 at the Right-hand End ſtands for 1000; the ſeveral Spaces and Sub-diviſions in the firſt Portion, are then to be eſtimated as were thoſe in the ſecond Radius, when the Right-hand End ſtood for 100; but now, in the ſecond Portion, the firſt Sub-diviſion in the Spaces ſtand for Hundreds, and the leſſer Diviſions of theſe ſtand for ſuch Parts of 100, as there are Parts introduced; thus where only two of the laſt Sub-diviſions can be introduced, each ſtands for 50; where 4 can be contained, each ſtands for 25; if 5 are put in, each ſtands for 20, and where 10 are introduced, each ſtands for ten.

In moſt Nautical Operations, where the Line of Numbers is uſed, the above mentioned Notation, or naming of the Diviſions, will be ſufficient; though Caſes may happen where the 1 in the Middle is to be taken as one; then the numbered Diviſions in the ſecond Radius ſtand for the Units between one and ten; the firſt Subdiviſions ſtand for tenth Parts of Unity, and the ſecond Subdiviſions ſtand for the Parts of tenths, according to the Number of theſe ſecond Sub-diviſions: And whatever Values are put upon any Diviſion or Sub-diviſion in the ſecond Radius, the Value of the correſponding Diviſion in the firſt Radius will be only a tenth Part.

Thus, if the Diviſion marked 5 in the ſecond Radius, ſtands for either 5000, or 500, or 50, or 5 tenth Parts, and ſo on; then the 5 in the firſt Radius will ſtand for 500, or 50, or 5, or 5 tenths, or 5 hundredths, and ſo on; and the ſame is to be underſtood of the other marked Diviſions, and alſo of the intermediate firſt and ſecond Sub-diviſions, every one of which ſtands for a tenth Part of the correſponding Diviſion and Sub-diviſions in the ſecond Radius.

This Line of Numbers is put as the middle Line on the Slider, and alſo on the lower Part of the fixed Scales.

To

To find a Diviſion, or Point on the Line of Numbers anſwering to any given Number, ſuppoſe 654.

Seek in the ſecond Radius between the Figures 6 and 7 for the 5th of the firſt Sub-diviſions; between this 5th and 6th are 4 ſecond Sub-diviſions, each of which anſwers to $2\frac{1}{2}$ Units; then will a Point taken a little nearer to the ſecond than to the firſt of theſe ſmaller Sub-diviſions, anſwer to the Number 654, ſufficiently exact for Uſe.

The correſponding Point in the firſt Radius, anſwers to the Number 65,4; if this Point in the firſt Radius anſwered to 654, the like Point in the ſecond, would anſwer to 6540.

The Point for the Number is 168, is thus found.

In the ſecond Radius between the Figures 1 and 2, ſeek the 6th of the larger Sub-diviſions, and the Point will fall between this 6th and the 7th of the Sub-diviſions, and the true Point will be on the 8th of the ſmaller ones, which will repreſent the Number 168. The like Diviſion in the firſt Radius repreſents 16,8, but if this ſtood for 168, then the like Diviſion in the ſecond Radius ſtands for 1680.

To find if the Line of Numbers be well divided.

Take with a Pair of Compaſſes the Diſtance between the Diviſions marked with the Numbers 1 and 2, this Diſtance will reach from 2 to 4, from 4 to 8, from 8 to 16, from 16 to 32, and from 32 to 64. This Diſtance alſo, will reach from $1\frac{1}{2}$ to 3, from 3 to 6, from 6 to 12, from 12 to 24, from 24 to 48, from 48 to 96; alſo, from $1\frac{1}{4}$ to $2\frac{1}{2}$, from $2\frac{1}{2}$ to 5, from 5 to 10, from 10 to 20, from 20 to 40, from 40 to 80; or the ſame Diſtance applied to any one of the Diviſions between 1 and 2, it will reach from thence to Diviſions found always by doubling the laſt Number. The ſame Work may be done without the Uſe of Compaſſes, by moving the Slider till the Left-hand 1 on the Line of Numbers ſtands againſt the Diviſion expreſſing any Number counted on the fixed Line of Numbers, then will the Diviſions, 2, 4, 8, 16, 32, &c. on the Slider, ſtand againſt reſpective Diviſions on the fixed Line, which will repreſent twice, four times, eight times, ſixteen times, &c. The Diviſion againſt which the 1 on the Slider was ſet. Theſe Diviſions on the fixed Line which correſpond to the Diviſions 2, 4, 8, 16, &c.

on the Slider, will be cut by the Edge of the Braſs Index, by moving it along to the Diviſions on the Slider.

To work Proportions on the Line of Numbers.

For this Purpoſe three Numbers are always given to find a fourth, which will eaſily appear, by ſtating the three given Terms or Numbers properly. For the ſame Extent of the Compaſſes which reaches from the firſt Term to the ſecond Term, will reach from the third Term to the fourth Term.

Or the Slider being moved till the Diviſion on it, taken for the firſt Term, ſtands againſt the ſecond Term taken on the fixed Line; then oppoſite to the third Term taken on the Slider, ſtands the 4th Term taken on the fixed Line, that is, bring the Edge of the Index againſt the ſecond Term taken on the fixed Line; and move the Slider till the firſt Term comes to the ſame Edge of the Index; then move the Index till its Edge cuts the third Term on the Slider, and the Edge of the Index will, at the ſame Time, cut the 4th Term among the Diviſions on the fixed Line.

Otherwiſe, the ſecond Term may be taken on the Slider, and ſet againſt the firſt Term taken on the fixed Line; then, againſt the third Term taken on the fixed Line, will ſtand the 4th Term taken on the Slider.

So that, If the firſt and third Terms are taken on the fixed Line, the ſecond and fourth Terms are to be taken on the Slider; and on the contrary, if the firſt and third Terms are taken on the Slider, then the ſecond and fourth Terms are to be taken on the fixed Line. The Slider may be drawn out either to the Right-hand or to the Left-hand, as it ſhall be found convenient to ſet the firſt and ſecond Terms oppoſite to one another.

EXAMPLE I.

Find a 4th Proportional to 3, 4, 9?

Now 3 on the Slider ſet againſt 4 on the fixed Line, againſt 9 on the Slider will ſtand 12 on the fixed Line, ſo 12 is the fourth Term.

EXAMPLE

EXAMPLE II.

If 6 ½ Yards coſt 22 Shillings, what will 26 Yards come to?

Now 6 ½ on the Slider being ſet to 22 on the fixed Line, then againſt 26 on the Slider, ſtands 88 on the fixed Line; ſo 88 Shillings, or 4l. 8s. is the Anſwer. Here the Slider is drawn out to the Left, till 6 ½ in the firſt Radius comes againſt 22 in the firſt Radius alſo on the fixed Line; then 26 in the ſecond Radius of the Slider will ſtand againſt 88 in the firſt Radius of the fixed Line.

EXAMPLE III.

If 352 Pounds of Goods coſt 128l. how much is that a Pound?

Move the Slider to the Right, till 128 in its firſt Radius ſtands againſt 352 in the firſt Radius of the fixed Line; then againſt the middle 1 of the fixed Line ſtands 364 nearly in the firſt Radius of the Slider.

Here at the firſt ſetting, the Left-hand 1 in both the fixed and ſliding Lines is called 100; afterwards, the middle 1 on the Slider Line is called one, conſequently the middle 1 on the Slider is alſo to be taken for one; therefore, the fourth Term 364, which on the Slider is to the Left of the middle one, is leſs than one, or is a Decimal Fraction, and muſt be reckoned as $\frac{364}{1000}$ *l.* and its Value is 7*s.* 3½*d.* for the Price of one Pound.

EXAMPLE IV.

If 1 ¾ groſs of Goods coſt 128 *l.* how much will 56 groſs come to?

Draw the Slider to the Right, till 128 on it ſtands againſt 1 ¾, or 1,75 in the fixed Line; then 56 on the fixed Line, will ſtand againſt 41 barely, on the Slider.

Here on the fixed Line, the Left-hand 1 is taken for one, therefore the 56 will be in the ſecond Radius: But on the Slider, the Left-hand 1 is taken as 100; conſequently the 41 nearly, in the ſecond Radius is 4100; but the Anſwer is 4096 *l.*

If theſe four Examples be well conſidered and underſtood, it will be found eaſy to ſolve other Queſtions of a like Kind; and alſo to eſtimate the Value of the fourth Term: But it is taken for granted, that a Knowledge of Decimal Fractions, and of the Rule of Proportion, or the Rule of Three, as it is uſually called, is familiar to the Operator; for where ſtrict Accuracy is wanted, it is beſt to perform the Work by the Pen. The Line of Numbers on the Gunter ſhould be uſed only as a Cheque on the higher Places of the Anſwer; the Units, or loweſt Place of a Number, conſiſting of 3 or more Places, being only to be eſtimated on theſe Lines, and ſo may be ſome Units too many or too few, as appears in the preceding 4th Example, where the 4th Term may be taken as 4100, when it is truly only 4096. And it muſt be remembered, that all Operations on theſe Logarithmic Lines, produce Reſults which cannot be eſtimated exactly; but are ſufficiently correct for moſt Nautical Purpoſes, particularly where the Courſe, Diſtance, or Difference of Latitude are concerned; and indeed, in moſt of the Aſtronomical Operations, where a Reſult within a few Minutes of a Degree does not produce any dangerous Conſequence.

The Lines of next Importance to theſe of Numbers, are the Lines of Sines and Tangents, for with theſe three Lines almoſt all the Caſes in Plane and Spherical Trigonometry, and conſequently in Aſtronomy, may be readily ſolved: And ſhould a greater Degree of Accuracy be required than is to be obtained by the Gunter, yet the Proportions being wrought by this Scale, will be a good Cheque to thoſe wrought by the Tables of Logarithms; for a wrong Figure uſed in a Logarithm, may produce a conſiderable Error in the Reſult, which may be readily detected by the Gunter; where the Operation is too ſmall to admit of any other Error than may ariſe in the Valuation of the ſeveral Sub-diviſions, with which, the Perſon who uſes this Inſtrument, ſhould be firſt well acquainted.

Of the Line of Sines.

The Tables of Logarithmic Sines, Tangents, Secants, and verſed Sines, are generally computed for a Circle whoſe Radius it 10,000,000.

Therefore, in the Sines the Index 9 belongs to all the Sines between

	90°	0′	0″	and	5°	44′	21″
And where the Index is 8, from	5	44	21	to	0	34	23
7, from	0	34	23	to	0	3	27
6, from	0	3	27	to	0	0	21

And ſo forth.

Now as the Length of the Gunter has only two Radius's on the Line of Numbers, containing ſuch Numbers only, where the Index in one Radius differs by Unity from the Index in the other Radius; therefore, within this Length no other Sines can be introduced, but ſuch whoſe Indices differ by Unity: And as not only the greateſt Number among the Sines, but alſo thoſe more generally wanted, have the Indices 9 and 8, differing by Unity; therefore, all the Sines between 90° and 0° 34′ are thoſe only which can be put on this Scale, the leſſer Sines extending far beyond this Length: And this is the Reaſon why the Sine of 90° has the ſame Termination with the End of the Line of Numbers.

The Space between 90° and 80°, is too ſmall to admit of more than five Diviſions; that neareſt to the 90 being the Mark for 85°; between 80° Degrees and 60°, the Diviſions are to ſingle Degrees; from 60° to 50° every Degree and half Degree are introduced; from 50° to 30° are Degrees and third Parts, or every 20 Minutes; from 30° to 20° are Degrees and Quarters, or every 15 Minutes; and from 20° downwards are put in the Degrees, and ſixth Parts of Degrees.

This Line of Sines is put the uppermoſt of the three on the Slider, and alſo the loweſt on the upper Part of the fixed Scales; whereby the two Scales of Sines are contiguous.

Of the Line of Tangents.

From the Property of Tangents it is known, that the Logarithms of Tangents and of their Complements, are Arithmetical Complements, one of the other; and conſequently will fall at equal Diſtances from 45 Degrees. Therefore, in the Line of Logarithmic Tangents, the Diviſions under 45 Degrees ſerve alſo for thoſe above, both being equally diſtant from the Diviſions choſen to repreſent 45 Degrees; which Diviſion therefore being a fixed Point, it is placed at the ſame Termination as the 90° on the Line of Sines, and deſcends as low as 0° 34′ as on the Sines, and for the ſame Reaſon, the Indices between the Limits of 45° and 0° 34′ being 9 and 8.

The Spaces between the Degrees from 10° to 45°, and back again from 45° to 80° are large enough to admit of four Spaces in each; therefore the Sub-diviſions of all the Degrees between 10° and 80 are to every 15 Minutes, but the Degrees under 10, and above 80, are each ſub-divided into 6 Parts of 10 Minutes each.

This Line of Tangents is, like the Sines, laid down twice on this Gunter, that is, once on the Slider, and once on the loweſt fixed Part; ſo that being in Contact, or Side by Side, the Reſult of Proportions worked by the Tangents may be aſcertained without the Uſe of the Braſs Index.

Of the Sine Rhombs, marked S. R. *and Tangent Rhombs, marked* T. R.

Beſide the two Lines of Sines and Tangents already deſcribed, which are laid down to Degrees and Parts of Degrees; another Line of Sines, and another Line of Tangents, are placed the two uppermoſt of the Lines on the fixed Part: But the Diviſions marked on theſe Lines belong only to the Degrees and Parts, which correſpond to the Arcs of the Horizon, meaſuring the Points and Parts of Points of the Rhombs in a Quadrant.

Thus to the Rhombs, or Points 1, 2, 3, 4, 5, 6, 7, anſwer the Arcs 11° 15′, 22° 30′, 33° 45′, 45° 0′, 56° 15′, 67° 30′, 78° 45′, and the Sines of ſuch Degrees and Minutes, being laid on the Sine Rhombs, will give the Diviſions anſwering to the whole Points; and the like for the intermediate Subdiviſions which are to every Quarter Point and half Quarter Point.

In the ſame Manner, the Diviſions on the Line of Tangent Rhombs, are taken from the Line of the Tangents of Degrees and Parts; ſuch as belong to the Degrees and Parts contained in the Rhombs, and Parts of Rhombs, wanted to be inſerted on the Scale.

Theſe Lines are particularly uſeful in working of Traverſes, or of finding the Amount of the Difference of Latitude and Departure from the ſeveral Courſes and Diſtances which a Ship has run in a given Time.

Of the Line of Verſed Sines, marked V. S.

Next under this Line of Tangent Rhombs, is placed the Line of Verſed Sines, beginning at 0 Degrees, and running thence towards the Left-hand to about 168 Degrees; one Extremity of this Line has the ſame Termination with the Numbers, Sines and Tangents, where the Mark for 180 Degrees ſhould be placed, but as the Scale begins at this Place, and the Diviſions may be ſuppoſed to go backwards, therefore 0 is put for the Beginning, and the Numbers thence proceeding through 10, 20, 30, &c. towards the Left, are only the Supplements of the Verſed Sines themſelves.

The Diviſions under 10° fall too cloſe to one another to be diſtinguiſhed.

From 10° to 60°, every ſingle Degree is introduced; from 60° to 100°, every Degree and half Degree; from 100° to 130°, every Degree and third of Degree, or 20 Minues; from 130° to 150°, every Degree and quarter Degree; and above 150°, every Degree and ſixth Part of a Degree.

Of the Scale of Longitude, marked Long.

This Line is at the Bottom of the fixed Scales, and is only a Line of equal Parts, repreſenting the Lengths of Degrees of Longitude, or of the Degrees taken on the Equator, in which the reckoning of a Ship's Longitude at Sea is kept; each Degree is about half an Inch, and ſub-divided in ten Parts of ſix Minutes each. From this Scale of equal Parts are the Meridional Degrees taken.

Of the Scale of Meridional Degrees, marked Mer.

This is a Line ſhewing the Length of the Degrees of Latitude, relative to the correſpondent Degrees of Longitude, reckoning from the Equator towards each Pole; the Degrees being gradually increaſed in the ſame Proportion as the Degrees of Longitude on the Globe diminiſh, from the Equator towards either Pole: That is, the Line is a Table of Meridional Parts applied to a Scale, where the firſt Meridional Degree is in

Length equal to an Equatorial Degree on the Scale of Longitude, and each of theſe Meridional Degrees is ſub-divided into ten Parts of ſix Minutes each.

This Projection of a Table of Meridional Parts takes in two Lengths of this Ruler; the firſt Length extends from the Beginning, at 0 Degrees on the Right, where it is marked *Mer.* and proceeds thence towards the Left as far as 50 Degrees; and begins again on the Right-hand at 50 Degrees, and goes on to the Left as far as 75 Degrees, which is as far as moſt Navigations extend, except the Greenland Fiſheries.

By theſe Scales a Mercator, or Wright's Chart for any propoſed Limits between the Equator and the Latitude of 75 Degrees, may be readily conſtructed, and of a Size ſufficiently large to lay down the Courſes made in any ſingle Day; but if a larger Scale ſhould at any Time be thought expedient, the ſame may eaſily be done with a very little Trouble, by accommodating the Meridional Parts to any larger Scale, as hereafter is ſhewn in the Uſe and Application.

Some Uſes to which this Inſtrument is applicable, but more eſpecially exemplified in what relates to Navigation.

And firſt, All Geometric Figures and Diagrams, both Plane and Spheric, of a propoſed convenient Size, may be conſtructed from the Natural Numbers or Lines on the Plane Scale, laid down on the reverſe Side of the Gunter; but it is thought rather unneceſſary to burthen this ſmall Treatiſe with a String of Problems relative to the conſtructive Part, as the ſame are to be met with in almoſt every Book of Navigation and Aſtronomy; I ſhall therefore proceed, ſecondly, to the Uſe and Application of thoſe Artificial Numbers or Lines on the Front of the Scale, diſtinguiſhed properly by the Name of Gunter.

To work a Proportion by the Sliding Gunter.

General Rule or Directions.

Let the Slider or moveable Part of the Rule be placed in ſuch Manner, that the firſt Term of the Proportion on the fixed Part, may be directly oppoſite to the ſecond Term in the Proportion upon the moveable Part; then againſt the third Term on the fixed Part, the fourth or Term required, will be found upon the ſliding or moveable Part. Or, the ſame Reſult will be found by taking the firſt and third Terms on the moveable, and the ſecond and fourth on the fixt Part.

Or, By the Scale with Compaſſes, take the following general Rule.

Firſt, Extend the Compaſſes from the firſt Term to the ſecond.

Second, That Extent laid the ſame way from the third Term, will reach to the fourth Term required.

PROBLEM I.

To perform Multiplication by the Line of Numbers.

Note, Multiplication is in Effect no other than the Rule of Three, or Proportion, by conſtituting Unity for the firſt Term.

Then, the Analogy will be, As 1 ·· Multiplier : : Multiplicand ·· Product.

Or, As 1 ·· Multiplicand : : Multiplier ·· Product.

EXAMPLE.

What is the Product of 15 multiplied by 6?

By the Slider.

Set the firſt Term 1, on the lower or fixt Piece, right againſt 15 the ſecond Term on the Slider, or middle Piece, then right againſt 6 on the fixt Piece, (always on the ſame Piece as the firſt) directed by the Braſs Index, is found 90 on the Slider, which is the fourth Term required.

Or with the Compaſſes.

Extend the Compaſſes from 1 to 15; that Extent laid the ſame way from 6, will reach to 90, the Product as before.

Hence it may be obſerved, that the firſt and chief preparatory Buſineſs is to digeſt and duly examine the true Import of the Queſtion, in order to obtain the proper Terms wherein the Proportion is to conſiſt, and this is called *Reaſoning*, which is founded upon a juſt Compariſon made of the Homogenials therein contained; and the Habitude or Relation which theſe Numbers or Quantities have to one another, when thus compared, is called the *Ratio*, (Reaſon) or Proportion; great Care then muſt be taken to form juſt Compariſons, for if theſe be true, the *Ratio* will be true, and the ſame will terminate in a true Reſult, otherwiſe it will be impoſſible. This being properly attended to in every Branch of Science, and the Proportion obtained, the foregoing general Rules will be ſufficient to point out, upon either Scale, the Mode of Operation, but to render the ſame yet more familiar and practicable, the following Problems (more immediately relating to Nautics) with the Proportions and Manner of Operation, are here preſented in the following Pages.

PROBLEM II.

To perform Diviſion by the Line of Numbers.

Diviſion is formed into the Rule of Three, by making Unity the ſecond Term.

It is to be noted, That though *Unity* neither increaſes or decreaſes the Reſult in Multiplication or Diviſion, yet when conſidered and taken as a Standard, or fundamental Figure, it is of very extenſive Uſe in Arithmetic and the Mathematics.

The Analogy or Proportion will then be, As the Diviſor ·· 1 : : the Dividend ·· Quotient.

EXAMPLE.

What is the Quotient of 96 divided by 6?

Directions by the Slider.

Set 6 the firſt Term, or Diviſor, on the lower or fixt Piece, againſt 1 the ſecond Term, on the Slide or middle Piece; then right againſt 96, the third Term or Dividend, on the fixt Piece (always on the ſame Piece as the firſt) directed by the Braſs Index, you will find 16 the fourth Term or Quotient required.

Or with Compaſſes.

Extend the Compaſſes from 6 to 1, that Extent laid the ſame way from 96, will reach to 16 as before.

PROBLEM III.

To reduce a Vulgar Fraction into a Decimal.

The Proportion is, as the Denominator of the given Fraction is to the Numerator : : 1 ·· Decimal Fraction required.

Note, A Decimal Fraction hath for its Denominator an Unit with as many Cyphers as its Numerator conſiſts of Places.

EXAMPLE.

Reduce $\frac{7}{8}$ into a Decimal Fraction ?

By the Slider.

Set 8 on the fixt Piece againſt 7 on the middle Piece or Slide; then againſt 1 on the firſt is, 875 on the Second, that is, $\frac{875}{1000}$, which is equal to $\frac{7}{8}$.

By Gunter, with Compaſſes.

The Extent from 8 to 7 laid the ſame way from 1, will reach to 875, as before.

PROBLEM IV.

Having two Numbers given, to find a third, fourth, fifth, &c. in continued or Geometrical Proportion.

The Analogy is, As the firſt Number ·· ſecond : : the ſecond ·· third : : the third ·· fourth : : the fourth ·· fifth, &c.

EXAMPLE.

Given 2 and 4, What are the third, fourth, and fifth Proportionals thereunto ?

Set 2 on the Slider to 4 on the outſide or fixt Piece, then againſt 4 on the firſt is 8 on the ſecond, for the third Proportional; and againſt 8 on the firſt is 16 on the ſecond, for the fourth Proportional; and againſt 16 on the firſt is 32 on the ſecond, for the fifth Proportional, and in like Manner proceed if more are wanted.

Or thus, with Compaſſes.

The Extent from 2 to 4 will reach from 4 to 8 for the third, and from 8 to 16 the fourth, and from 16 to 32 the fifth continued Proportional, &c.

PROBLEM V.

Having three Numbers given to find a fourth, that ſhall bear ſuch Proportion to the third, as the ſecond does to the firſt.

This is called interrupted Proportion, or the Rule of Three.

Note, That if four Numbers be in Proportion continued or interrupted, the Rectangle or Product of the two Extreams, is equal to the Product of the two Means.

The Analogy is, As the firſt Number ·· the ſecond : : the third ·· fourth, Or,

As the firſt ·· the third : : ſecond ·· fourth.

EXAMPLE.

If the Diameter of a Circle be 28, what is the Circumference ?

Archimede's Proportion of the Diameter to its Circumference, is as 7 to 22, which is ſufficiently exact for Operations by Gunter; therefore it will be,

As 7 ·· 22 : : 28 ·· 88, Or,

By the ſliding Scale.

Set 7 on the Slider againſt 22 on the fixt Piece; then againſt 28 on the firſt is 88 on the ſecond, the Circumference required.

Or, againſt 7 on the outſide ſet 22 on the Slider; then againſt 28 on the firſt will be 88 on the laſt.

Hence it appears, that whenever practicable, you may take your firſt Number on the Slider or fixt Piece, and draw the Slider out to the Right-hand, or to the Left, as ſhall be found moſt convenient, but great Regard muſt be had to your Notation, in order to eſtimate the true Value which the Numbers muſt bear in each reſpective Radius.

By the Compaſſes.

The Extent from 7 to 22, laid the ſame way from 28, will reach to 88, as before.

PROBLEM VI.

Having three Numbers given to find a fourth in duplicate Proportion.

This is chiefly uſed in the Compariſon of Lines with Superficies: Wherein obſerve, that the firſt and ſecond Terms muſt be of one Kind, or Denomination.

EXAMPLE.

What is the ſuperficial Content of a Circle, whoſe Diameter is 28?

If the Diameter of a Circle be 1, the ſuperficial Content, or Area will be 0.78539.

Therefore, As Diameter 1 ·· 28 the Diameter : : 0.785 Area of 1 ·· fourth and : : fourth to the required Area.

By the Slider.

Bring 1 on the middle Piece, againſt 28 on the fixt Piece; then againſt 0.785 on the firſt, is 22 on the ſecond, and againſt 22 on the firſt is 616 on the ſecond, for the Area.

By the Scale with Compaſſes.

The Extent from 1 to 28 will reach from 0.785 to 22; and the ſame Extent laid the ſame way from 22, reacheth to 616, as before.

In both the above, during the Courſe of the Extenſion, it is found neceſſary to alter the Value of the Notation Number in its proper Radius, great Care then ought to be taken that a proper Value be announced upon the Reſult; a little Conſideration and Practice, will render the whole of this intelligible and familiar.

PROBLEM VII.

Having three Numbers given to find a fourth in triplicate Proportion.

This relates to the Compariſon of Lines with Solids, in ſtating which Proportions it muſt be obſerved, that the firſt and ſecond Terms are always to be of one Denomination.

EXAMPLE I.

If an Iron Bullet of 4 Inches Diameter, weigh 9 Pounds, what is the Weight of another of the ſame Metal, whoſe Diameter is 6 Inches?

The firſt and ſecond Terms muſt, as before, be both Lines, that is, the given Diameters, and the Proportion will ſtand

Thus, as Diameter 4 ·· Diameter 6 : : Weight 9 ·· 13.5, and : : 13.5 ·· 20.2, and : : 20.2 ·· 30.3 Pounds Weight required.

By the Slider.

Set 4 againſt 6, and againſt 9 on the firſt is 13.5 on the ſecond; and againſt 13.5 on the firſt, is 20.2 on the ſecond; and againſt 20.2 on the firſt, is 30.3 on the ſecond, the Weight.

By Compaſſes.

The Extent from 4 to 6 being laid on three times from 9, will reach to 30.3 Pounds as before.

EXAMPLE II.

If a Gun of 6 Inches Bore require 11 Pounds of Powder, how much will ſerve a Gun of 4 Inches Bore ?

The Proportion is, As 6 ·· 4 : : 11 ·· 7.3, and : : 7.3 ·· 4.88, and : : 4.88 ·· 3.25 the Pounds required.

PROBLEM VIII.

To extract the Square and Cube Roots by the Line of Numbers.

EXAMPLE.

Extract the Square Root of 64 thereby, and alſo the Cube Root of the ſame Number.

Take in your Compaſſes one-half of the Diſtance between 1 and 64, which half Extent will juſt reach from 1 unto 8, which is the Square Root required——And if you take the one-third of the Diſtance in your Compaſſes, between 1 and 64, that Extent laid off from 1, will reach exactly from 1 to 4, which is the Cube Root required ; this Line is no other than a Line of Logarithms, and any Log. divided by 2, quotes the Log. of the Square Root of its natural Number ; if divided by 3, it gives the Log. of the Cube Root, and if by 6, it gives the Log. of the Root of the ſixth Powers, &c.

Hence the Roots of all Powers may eaſily and readily be found on the Line of Numbers, by dividing the Diſtance or Extent on the Scale, between Unity and the Number given to be extracted, into ſo many equal Parts as are directed by the Index of that Power, that is, the Square or 2d Power into two equal Parts ; the Cube or 3d Power into three equal Parts ; the Biquadrate or 4th Power, into four ; the Square cubed, or 6th Power, into 6, &c. which half, third, fourth, and ſixth, &c. Part, taken in the Compaſſes, and laid off from Unity (obſerving well the Notation, or Value of the Figures) will point out or give the reſpective Root required.

This is very practicable and familiar with the Compaſſes, but theſe may not always be at Hand, and the Intention of this new Scale, is to perform as much as poſſible, without Compaſſes ; what then muſt be done ?

This is not an improper Queſtion, and therefore to ſupply this apparent Defect, and ſatisfy the curious Enquirer, obſerve the following Directions; and in the firſt Place it is to be noted, that you have an Equatorial Line, or Line of equal Parts at the Bottom of the Scale, connected with the Lines of Meridional Parts, and extending from 0 at the Right-hand to 58 at the Left; this Line, with the Log. Line of Numbers, and the Braſs Index, will do the Buſineſs.

Firſt, Set the Edge of the Braſs Index or Director to 58 at the Left-hand, move the Slider until 1, on the Log. Numbers ſtands in a Line under the Edge of the ſaid Index, turn the perpendicular or vertical Screw, and there fix or ſecure the Slider in the Braſs Socket; this may be called the Rectification of the Scale or Slider.

Then, Note, on the Line of Log. Numbers, that Point which repreſents the Number given to be extracted, or whoſe Root is required, in this Example, 64; move the Edge of the Braſs Index to 64, and note, what Number and tenths on the aforeſaid Line of equal Parts anſwer thereunto, in this Caſe, 6.5 from the Right-hand, or properly 51.5 equal Parts from the Left-hand at 58, for this will be the proper Diſtance of 64 from Unity on the Log. Line of the Numbers and this Diſtance (51.5) will be the Compliment of 6.5 to the whole Line 58; then for the Square Root, take half of this Diſtance, viz. 25.75, move the Index to 25.75 on the ſaid Line of equal Parts, reckoning backwards from 58, or to 32.25, its Complement to 58, reckoning forwards from the Right, and directly againſt 32.25 on the Log. Line of Numbers on the Slide, you will find 8 for the Square Root, as before. And for the Cube Root, divide 51.5 found as above, by 3, that is, take one-third thereof, which is 17.17 nearly, ſlide the Edge of the Index to 17.17 on the Line of equal Parts, reckoning backwards from 58, or forwards to 40.83 its Complement to 58, and you will have in a Line therewith on the Line of Log. Numbers 4 for the Cube Root; in like Manner proceed for any other Number or Root, duly regarding the Notation.

I ſhall preſume upon the Reader's Patience for one Example more in an higher Power.

EXAMPLE.

What is the Square Cube Root of 64, or the ſixth Power?

The Scale being rectified as before directed; ſet the Index at 64 upon the Line of Log. Numbers, and againſt it, you will find 6.5 reckoning forwards, or 51.5 reckoning backwards from 58 on the Line of equal Parts; divide this Number by 6, becauſe the Square Cube is the 6th Power, and the Quotient or 6th Part is 8.583+, the Complement of which to 58 is, 49.416+, move the Index to 49.416+, and right againſt this Number on the Line of Log. Numbers under the Edge of the Braſs Index, you will find 2, which is the Square Cube Root required.

Note 1. The Slider may be drawn quite out, and inverted; for it will, in this Poſition, move ſmoothly through the Socket and Grooves, and then the Numbers or Figures on the Log. Numbers, will increaſe from Right to Left, in the order of the equal Parts, but they will then ſtand upſide down on the Slider, which may appear ſomewhat aukward, but the Practitioner may herein do as he pleaſes. But,

Note 2. That all this Trouble of reckoning backwards, or taking the Complements to 58, for the more readily reckoning forwards, may be ſaved, if a ſmall Sub-margin be left under the Line of equal Parts, and the ſame primary Graduations or Diviſions be marked therein with, 0, 1, 2, 3, 4, &c. from 58, towards the Right-hand encreaſing (the Subdiviſions or tenths being the ſame when taken either way) for then the Point or Numbers thereon by the Edge of the Index, will ſtrike the Eye immediately, and that without riſque of Error, or further Trouble.

It is hoped that the foregoing Directions will ſufficiently explain the Uſe and Application of this eſtimable Line, I ſhall therefore juſt touch upon Superficies and Solids, with their Proportions only, in order to introduce the Meaſuration of Bales, Caſes and Shipping, ſo neceſſary for a Mariner to underſtand, and then proceed to the Uſe of the other Lines in Conjunction with, or without *this*, in Nautical Buſineſs.

PROBLEM IX.

Of SUPERFICIES.

The Length and Breadth of any Square, or Parallelogram given, to find the Area, or ſuperficial Content thereof.

The Proportion is, As 1 ·· the Breadth : : Length ·· Content.

PROB.

PROBLEM X.

Of SOLIDS.

The Length, Breadth and Depth of any Square Solid, or Parallelopipedon being given, to find the Content?

The Proportion is, As 1 .. the Breadth : : Depth .. fourth Number, And

Then, As 1 .. fourth Number : : Length .. Content, in the ſame Kind of Meaſure with the Dimenſions given.

Note, That 12 Inches make one Foot in Length; a ſuperficial Foot contains 12 times 12, or 144 Inches; and a ſolid Foot 12 times 144, or 1728 Cubic Inches.

PROBLEM XI.

To find the Tunnage of a Box, Bale or Caſe, having its Length, Breadth and Depth given.

The Proportion is, As 66 .. Breadth : : the Depth .. fourth Number:

Then, As 1 .. fourth Number : : Length .. Tunnage required.

Note, That 66 Feet is the Content of a Caſe that will incloſe two Engliſh Butts, but the Cantlings of them being better than one-third Part; therefore allowing 26 Feet for the Cantlings, the remaining 40 Feet is counted to be one Tun.

And then the Proportion will be, As, 40 .. the Breadth : : the Depth .. fourth Number.

Then, As 1 .. fourth Number : : the Length .. Tunnage.

EXAMPLE.

A Caſe or Bale being 6 Feet broad, 4 Feet deep, and 10 Feet and an half long; what is the Tunnage?

By the Slider, or with the Compaſſes.

As 40 .. 6 : : 4 .. 0.6 the fourth Proportional;
And, As 1 .. 0.6 : : 10.5 .. 6.3, or 6 Tuns and $\frac{3}{10}$ Part of a Tun, the Content required.

PROBLEM XII.

To meaſure the Tunnage of Ships, according to the Cuſtom or Practice of Ship-builders about London.

RULE.

RULE.

Multiply the Length of the Keel, Breadth of the Beam, and the half Breadth into one another, and divide the laſt Product by 94, gives the eſtimated Tunnage. — Hence will ariſe theſe Proportions for the Scales, viz.

As 94 ·· Breadth : : the half Breadth ·· fourth Number.
Then, As 1 ·· fourth Number : : the Length ·· Tunnage.

But if you rather chooſe to have the whole Breadth inſtead of the half Breadth, for your third Term in the firſt Proportion, this may be done by doubling the Diviſor, or firſt Term 94, and then it will ſtand thus :

As 188 ·· Breadth : : the Breadth ·· fourth Number,
And, As 1 ·· fourth Numb. : : the Length ·· Tunnage.

EXAMPLE.

A Ship 72 Feet by the Keel, and 24 Feet by the Beam, what is her Tunnage ?

As 94 ·· 24 : : 12 ·· 3.06. And as 1 ·· 3.06 : : 72 ·· 120.3 Tuns
Or,
As 188 ·· 24 : : 24 ·· 3.06. And as 1 ·· 3.06 : : 72 ·· 120.3 as bef.

The Uſe of the ſundry Lines upon *Gunter's* Scale, deduced from Trigonometry, and applied to the Buſineſs of Navigation.

Plane Sailing by ſingle Courſes.

PROBLEM XIII.

Courſe and Diſtance given, to find the Difference of Latitude and Departure from the Meridian.

EXAMPLE.

Suppoſe a Ship ſails 496 Miles, S.W. by W. from the Lizard, what Difference of Latitude and Departure has ſhe made ?

The Proportions ariſing from Trigonometry, are theſe,

As Radius ·· Diſtance : : Sine Co. Courſe ·· Diff. Latitude.
S. 90° ·· 496 M. : : S. 33° 45′, or 3 Pts. ·· 275.6 Miles.

And as Radius ·· Diſtance : : S. Courſe ·· Departure.
S. 90° ·· 496 : : S. 56° 15′ ·· 412.4 Miles.
Or 5 Points.

Directions. Right againſt 90 Deg. or Radius on the Line of Sines, or 8 Points on the Line of Sine Rhombs on the outſide Piece, ſet 496 on the Slide, or middle Piece; then againſt 33° 45′ on the Line of Sines, or, which amounts to the ſame, 3 Points on the Line of Sine Rhombs on the outſide, or fixt Piece, is 275.6 Min on the Slider, for the Difference of Latitude, and againſt 5 Points or 56° 15′, on the Sines, is 412.4 Minutes, the Departure.

With Compaſſes.

The Extent from 8 Points, or 90 Deg. on the Line of Sine Rhombs, or the Line of Sines, to 496 Min. on the Line of Numbers will reach the ſame way, from 3 Points on Sine Rhombs, or 33° 45′ on Line of Sines, to 275 6 on Line of Numbers, the Difference of Latitude; and the Compaſſes being kept at the ſame Diſtance, will reach (the ſame way) from 5 Points, or 56° 15′, to 412.4, the Departure, as before.

Note 1. That Regard muſt be had to the Value of the Subdiviſions upon the Lines of Sines and Tangents, which are to be eſtimated by the Number of them contained between each Degree.

Note 2. In order that each Proportion may (if poſſible) ſtand in one Line, the Radius, according to the Nature of the Proportion, may be any of theſe, viz. 8 Points on the Line of Sine Rhombs, 4 Points on the Tangent Rhombs, 90 Degrees on the Line of Sines, 45 Degrees on the Tangents, or, 0 on the verſed Sines.

Note 3. The two Proportions before given, do ariſe Trigonometrically by making the Hypothenuſe, or Diſtance in the Right-angled Triangle, to be the Radius of a Circle; but two other Proportions may be deduced to find the ſame Thing, by making each of the Legs Radius; an Inſtance of which ſhall be given here for Variety ſake, to find the Departure.

And firſt, by making the Perpendicular or Difference of Latitude to be the Radius.

1. Then it will be,

As, Secant of Courſe ·· Diſtance : : Tangent of Courſe ·· Departure.

E Whence

2. By making the Baſe, or Departure to be the Radius.

Whence this Proportion,

As, Sec. Comp. Courſe ·· Diſtance : : Radius ·· Departure, which will produce 412.4 as before.

Now theſe Proportions are eaſily worked by a Table of Logarithms, but a Queſtion naturally ariſes in this Place, viz. How are theſe (and others ſuch as theſe) to be ſolved by Gunter, as there is no Logarithmic Line of Secants thereupon? It is true there is none, it is therefore a proper Queſtion; but this Defect may be ſupplied without the Addition of a new Line, which I ſhall endeavour to explain by firſt conſidering one Property belonging to the Tangents, which is, that Radius is a mean Proportional between the Tangent and Co-Tangent of any Arc; hence, the Complement Arithmetical of Log. Tangent of an Arc, with its proper Index, is the Log. Co-Tangent of the ſame Arc, and for this Reaſon Tangents above 45 Degrees are reckoned from Radius upon the ſame Diviſions, encreaſing backwards, or towards the Left-hand, that is, the Diviſion 40 forwards, becomes 50 backwards, being equidiſtant from the Radius; the 30 forwards now becomes 60; and in like Manner muſt all the leſs Diviſions and Sub-diviſions be eſtimated, and this is done to prevent an awkard encreaſe of the Length of the Scale; but in Operations, the Point repreſenting any Tangential Degree above 45, muſt be conſidered as ſtanding, or being juſt as much beyond 45 to the right, as the Extent of that *Exceſs* is ſhort of 45 upon the Scale: This recurrence of the Tangents above Radius may, perhaps at firſt ſight, appear ſomewhat puzzling in the Operations, but if this Explanation be duly digeſted and underſtood, the apparent Difficulty will ſoon vaniſh, and a Method of performing all ſuch Operations on either Scale, with the true Reaſons thereof, will ſoon preſent itſelf to an ingenious Mind; but as Example tends to eſtabliſh, and is ſometimes better than Precept, I ſhall here preſent you with two or three.

EXAMPLE I.

Suppoſe a Ship's Difference of Latitude is 275, and the Departure 412, what is her true Courſe?

The Proportion is,

As Diff. Lat. ·· Departure : : Radius ·· Tangent Courſe.
275 ·· 412 : : T. 45° ·· T. 56° : 15'.

By the Slider.

Set 275 Num. on the Slide right againſt 412 Num. on the fixt Piece, by the help of the Braſs Index, then will you have Radius, or 45 Tang. on the firſt, or Slide, ſtand without, or beyond the fixt Radius, at the end of the Scale, and was that fixt Tangent Line continued forwards inſtead of backwards, the Point repreſenting 56° : 15′ on the ſame would ſtand directly againſt it, that is, againſt the Radius on the Slide; and to know upon every Occaſion, without an Index continued, what Extent you have beyond the Radius, you need only count backwards; for Radius being a mean Proportional between the Tangent and Tangent Complement, the ſame Log. Diſtance or Extent you have beyond Radius, will determine the Degree or Point required, being eſtimated to the left, or ſhort of Radius. In this Caſe therefore, having adjuſted the firſt and ſecond Terms as afore directed, bring the Braſs Index to the Radius on the fixt Piece, and in a Line therewith, lies 56 : 15 on the Tangent, or 5 Points on the Tangent Rhombs, reckoning from Radius to the left on the Slide.

Note, If the ſecond Term be greater than the firſt, then the fourth, or Term required, will be greater than the third; in this Caſe more than 45 Degrees.

The ſame with Compaſſes.

It may be eaſily ſeen, by the Note above, that the Term required will be greater than Radius, and becauſe the Tangents above 45 recur, invert the Proportion,

Thus, As 412 ·· 275 : : T. 45° ·· T. 56° : 15′, or 5 Points.

The Extent of the Compaſſes from 412 to 275, wlll reach, laid the ſame way, from Tang. 45 Deg. or 4 Points on the Tang. Rhombs, to T. 56° : 15′, or 5 Points, on Tang. Rhombs, eſtimated towards the left increaſing.

Note, That the two following Proportions do not ariſe from the preceding Problem, or any Part of that which is now immediately before us, but are here introduced (for the Variety therein contained) to illuſtrate more fully the mode of Tangential Operation.

EXAMPLE II.

As, S. ½ Diff. of 2 Sides ·· S. ½ Sum : : Tang. ½ Diff. of two Angles ·· Tang. Comp. ½ the contained Angle.

S. 15° : 36′ ·· S. 49° : 43′ : : Tang. 52° : 2′ ·· Tang. 74° : 37′.

By the Slider.

Set S. 15° : 36′ on the Slide right againſt S. 49° : 43′ on the outſide Piece; then in a Line, directed by the Braſs Index, over Tangent 52° : 2′ on the Slide, you will have 74° : 37′ on the Tangents upon the outſide, or fixt Piece.

As both the third and fourth Terms here are above 45°, the Work is as uſual, only that you muſt reckon them both from Radius to the left encreaſing.

The ſame with Compaſſes.

Extend from S. 15° : 36′ to S. 49° : 43′; this Extent laid from T. 52° : 2′ the contrary way, or towards the left, will reach to T. 74° : 37′, as before.

But if you invert the Proportion, and ſay, As S. 49° : 43′ the ſecond Term, is to S. 15° : 36′, the firſt, the whole of the Extenſion will be from the right to the left on the Scale, and the Reſult the ſame.

In the foregoing Example, both the two laſt Terms, Tangents, are more than 45 Degrees; it may be proper therefore to give an Example wherein one of the laſt Terms (Tangents) ſhall be greater than Radius, and the other leſs.

EXAMPLE III.

S. co. ½ Sum of 2 Angles ·· S. co. ½ their Diff. : : T. ½ contained Side ·· T. ½ Sum of 2 Sides.

S. 10° : 28′ ·· S. 37° : 58′ : : T. 19° : 14′ ·· T. 49° : 45′.

By the Slider.

Set S. 10° : 28′ on the Slide, by the Edge of the Braſs Index in a Line with S : 37° : 58′ on the outſide or fixed Piece, then will S. 19° : 14′ on the Slide, extend beyond the Radius; wherefore ſore note what Deg. and Min. on the Tangent Slide, ſtands againſt T. 45, (which in this Caſe is T. 16 Deg. 30 Min. nearly) then remove the Slide until the third Term, 19° : 45′ ſtand againſt Radius, and directly oppoſite to T. 16° : 30′ on the Slide, you will meet with T. 49° : 45′ on the fixt or outſide Piece.

The ſame with Compaſſes.

Extend from S. 10° : 28′ to S. 37° : 58′, this Extent will reach beyond the Radius; wherefore lay this Extent from T. 45 toward the left, and note what Degree the Left-hand Point falls upon, (which here will be T. 16° : 30′ as before) then the Extent of the Compaſſes from this Point to T. 19° : 14′, being laid reverſely from T. 45°, will reach to Tang. 49° : 45′, the fourth Term required, as before.

Theſe Examples familiarized by a little Practice, and the Explanation before given duly conſidered, will be ſufficient to elucidate the whole of this curious Buſineſs, and enable the Student to underſtand what Analogy the Tangents bear to Radius, and to one another, and conſequently to diſpoſe the Terms, and perform any Operation upon either Scale, with Propriety, wherein Tangents with Numbers, or Tangents mixed with Sines are employed. Having thus exemplified the Manner of working by Log. Tangents, and the Reaſon of the Operations, I ſhall now proceed to ſhew how any Proportion with Secants combined, may be performed upon either Scale, as the ſame are now conſtructed, without any Addition.

In order to explain this, it muſt be noted as a Fundamental, That Radius is a mean Proportional between the Sine of any Arc or Angle, and the Secant Complement of the ſame Arc; or, between the Secant of any Arc, and the Sine Complement of the ſame; hence, the Complement Arithmetical of the Log. Sine of an Arc, with its proper Index, is the Log. Co. Secant of the ſame Arc, and inverſely, the Co. Ar. of the Log. Secant of an Arc, with proper Index, is the Log. Co. Sine of the ſame. And hence it evidently appears that a Line of Log. Secants is no other than a Line of Log. Sines inverted; and another Line of Sines marked 0, 1, 2, 3, or 10, 20, 30, the contrary way, and placed at the End, or at 90 Degrees, that is, the 0 at 90, &c. will compleat a Line of Secants, and this is the very Caſe with the ſaid Lines on the Natural Scale, for thereupon where the Sines end at 90, there the Secants begin: The ſame Line of Log. Sines then, may be called Sines forwards, and increaſing from the Left-hand to the Right, and Secant Compliments backwards, and reckoned from the Radius, or 90, increaſing towards the left, that is, the Sine of 80° will be the Secant of 10°; Sine of 70° will be the Secant of 20° its Compliment, &c. An Example or two, with the Manner of Operation upon both Scales, will make this Plain; for which Purpoſe I ſhall refer back to the Varieties given under Problem XII. where the two Legs are reſpectively made Radius, And,

Firſt, for the Perpendicular, or Diff. Latitude:

As Sec. Courſe ·· Diſtance : : Tang. Courſe ·· Departure.
Sec. 56° : 15′, or Sec. 5 pts ·· 496 m. : : Tan. 56° : 15′, or Tan. 5 pts ·· 412.4 m.

Note, In all Proportions whatſoever, but more eſpecially in ſuch as theſe, it is to be carefully attended to, whether the Term required will be greater or leſs than the ſecond of the ſame Denomination, and this is known by comparing the firſt with the third, for if the third be greater than the firſt, the fourth will be greater than the ſecond, and the contrary; now in this Caſe, though the Number of Degrees in the firſt and third be equal, yet a Tangent of any Angle will be ſhorter or leſs than the Secant of the ſame Angle, ariſing from the very Properties of Tangents and Secants, therefore the fourth will be leſs than the ſecond, and as the Secants upon the Scale, and alſo the Tangents above 45, are reckoned reverſely, or towards the left thereupon, this Proportion muſt be inverted, or ſtand thus, viz. As the third Term is to the firſt, ſo is the ſecond to the fourth Term required; or, as the third is to the ſecond, ſo is the firſt to the fourth.

By the Slider.

Set Tang. 56° : 15′, or T. 5 pts. on the fixt Piece, by the Help of the Braſs Index, in a Line with Sec. 56° 15′, equal to 5 pts. on the Slide, reckoned backwards from 90° on the Sines; then move the Index to 496 Num. on the fixt Piece, and you will find 412.4 Min. on the Slide, for the fourth Term required.

With the Compaſſes.

Extend the Compaſſes from Tang. 5 pts. or 56° : 15′, to Secant 5 pts. or 56° : 15′ reckoned backwards on the Sines, (which is Sine of 3 pts. forwards) this Extent, laid the ſame way, will reach from 496 Min. on the Line of Numbers, to 412.4 Min. as before.

Variety ſecond, When the Baſe, or Departure is made Radius.

Then. As Sec. Co. Courſe ·· Diſtance : : Radius ·· Departure.
Sec. 3 Pts. or 33° 45′ ·· 496 M. : : Secant 0 ·· 412.4 M.

It is here apparent that the third Term, or Radius, is leſs than the firſt Term, or a Secant of 3 Points; therefore the fourth Term will be leſs than the ſecond, or 496 Min. and this would turn out ſo as the Proportion now ſtands, if we had a Line of Secants tacked to the End of the Sines, but as the Line of Sines are now to be reckoned Secants inverted, the Proportion muſt be inverted, and then it will ſtand thus, viz. As the third Term is to the firſt, ſo is the ſecond to the fourth.

By the Slider.

Set Radius, or Secant 0, on the fixt Piece, by the Index, againſt Secant 3 Points, or 33° : 45′ on the Slide, then right againſt 496 Min. on the Line of Numbers on the fixt Piece, will ſtand 412.4 Min. on the Line of Numbers on the Slide, as before.

With Compaſſes.

Extend from Radius, to the Secant of 33° : 45′ or 3 Points, reckoning from S. 90; this extent, laid the ſame way from 496 on the Numbers, will reach to 412.4 Min. as before.

Note, That if you draw the Slider out of the Grooves and Socket, and invert it, ſetting 90° on the Slider againſt Radius on the fixt Piece, and faſten it there with the Screw; then will the Line of Sines on the Slider be a Line of Secants, reckoning 10, 20, 30, on the Line of Sines for 80, 70, 60, &c. as Secants: And the Line of Tangents on the Slider, thus placed, will become a Line of Tangents properly continued from 45, and increaſing to 50, 60, 70, &c. towards the right, as the ſame are now marked and reckoned reverſely towards the left from Radius; but by this Diſpoſition the Scale being extended to near five Feet, is therefore unhandy and troubleſome.

I have endeavoured herein to be as explicit as poſſible, and it is hoped that the Reader will hereby be ſufficiently enabled to underſtand and perform any Operation upon the Scales wherein Sines, Tangents, or Secants are concerned, and have dwelt rather longer upon the Subject, as I do not recollect to have ſeen it treated of ſo fully elſewhere.

PROBLEM XIII.

To reſolve a Traverſe by the Scales.

EXAMPLE.

A Ship in 40 Degrees North Latitude, and 5° : 14′ Weſt Longitude, ſails firſt S.E. by S. 68 Min. then S.W. by W. 55 Min. and then W.N.W. 75 Min. What is her Difference of Latitude and Departure in this general Run?

The Solution of this Problem depends intirely upon the laſt, and is of great Utility in the Theory of Navigation, for the Courſe and Diſtance (obtained by the Compaſs and Log.) may be eſteemed as Elements or Principles upon which the conſequent Operations are Trigonometrically founded, and if theſe were ſtrictly true, the Latitude and Longitude of the Ship would be truly known at any Time; but theſe two Principles are ſubject to Errors, ariſing from various Cauſes, to correct which by the moſt proper and practicable Methods, the Skill and Induſtry of the moſt able Mathematicians have been employed; however, we are obliged in the firſt Inſtance, to make Uſe of the Courſe and Diſtance; great Care then ſhould be taken that theſe be as true as poſſible; at the ſame Time we cannot well reject the Uſe of Plane Sailing, which is erroneous, for the Earth and Sea do not form a flat Surface, but a Globular, yet in very ſhort Diſtances (ſuch as in the above Example) the Error will be inconſiderable, provided a Correction be made, by Meridional Parts, at the End of every twenty-four Hours: This Problem then, and that preceding, become preparative, and very neceſſary ones, as the Difference of Latitude and Departure for each reſpective Courſe and Diſtance muſt be firſt found, and then collected, and ſo diſpoſed as to make out readily, and with Convenience, the Difference of Latitude and Departure made, during the Space of this 24 Hours; a Specimen of which Diſpoſition is given in the ſmall Traverſe Table below.

To find the ſeveral Differences of Latitudes and Departures by a Table of Logarithms, takes up more Time than can be well ſpared at Sea, unleſs the Weather be ſerene, and a Table of Difference of Latitude and Departure are not always at Hand; this Scale is, therefore, a moſt uſeful Inſtrument in this Caſe, for Diſpatch, and is ſufficiently exact, as the Diſtances never extend to more than three Places; the Courſes being alſo given in Points and Quarters only, the ſame are readily diſtinguiſhed, and catched by the Eye, on the Rhomb Lines.

The Proportions are, for

First Courſe.

As Rad. ·· Diſt. : : S. Co. Cour. ·· Diff. Lat. : : S. Cour. ·· Depar.
S. 8 pts. ·· 68m. : : S. 5 pts. ·· 57 m. S. : : S. 3 pts. ·· 38m. E.

Second Courſe.

S. 8 pts. ·· 55m. : : S. 3 pts. ·· 30m. 6 S. : : S. 5pts. ·· 46m. W

Third Courſe.

S. 8 pts. ·· 75m. : : S. 2 pts. ·· 28.8m. N. : : S. 6pts. ·· 69.5 W.

Traverſe Table.

			Diff	Lat.	Departure	
	Courſes.	Diſt.	N.	S.	E.	W.
1	S.E.by S.	68		57.0	38.0	
2	S.W.b.W	55		30.6		46.0
3	W.N.W.	75	28.8			69.5
			28.8	87.6 28.8	38.0	115.5 38.0
	general. Diff. Lat.			58.8		77.5

Departure made good in this Day's Run.

To work the above on the Sliding Gunter.

Set Sine of 8 Points on the fixt Piece, by the Help of the braſs Index, againſt 68 Num. on the Slide; then draw the ſaid

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Index to S. 5 Points, you will find 57 Num. on the Slide, for the Difference of Latitude, (and without any Alteration) againſt S. 3 Points, on the fixt Piece, you will have 38 Num. on the Slide, for the Departure.

With the Compaſſes.

Extend from 8 Points on the Sine Rhombs to 68 on the Numbers; the ſaid Extent laid the ſame way from Sine 5 Points, will reach to 57 on Num. and the ſame Extent laid the ſame way, will reach to 38 on the Num. as before; and in like Manner may the reſt, and all others of the like Kind, be wrought with Eaſe and Diſpatch.

PROBLEM XIV.

Difference of Latitude and Departure from the Meridian given; to find the Courſe, Diſtance, and Difference of Longitude, by middle Latitude Sailing.

EXAMPLE.

By the Reſolution of the Traverſe in the laſt Problem, it appears that the Difference of Latitude made good in the Day's Run, was 58.8 Miles S. becauſe the S. Column is greateſt; and the Departure 77.5 Miles Weſterly, becauſe the Weſt Column is greateſt.

The Ship's direct Courſe, Diſtance, and Longitude in, are then required?

Firſt, By making the Perpendicular, or Difference of Latitude to be Radius.

The Proportion will be,

As, Diff. Latitude ·· Radius : : Departure ·· Tan. Courſe.
58.8 Min. ·· T. 45° : : 77.5 Min. ·· Tan. 52° 50′ from the S° towards the Weſt, equal to $5\frac{3}{4}$, Pts. or S.W. $\frac{3}{4}$ W. almoſt.

Second, By making the Baſe or Departure the Radius, then the Proportion will be,

As Departure ·· Radius : : Diff. Lat. ·· T. Co. Courſe.
77.5 M. ·· T. 45 : : 58.8 M. ·· T. 37° 10′.

On the Slider, for the firſt.

Becauſe the third Term is greater than the firſt, and the ſecond a Tangent 45, therefore the fourth will be more than 45, and the Proportion worked reverſely; therefore ſet T. 45 on the fixt Piece, againſt 77.5 Num. on the Slide, then againſt 58.8 on the Slide, you will have 52° : 50′ on the fixt Tangents, for the Courſe, reckoning backwards.

With Compaſſes, for the firſt.

The Extent from 77.5 to 58.8 on the Line of Numbers will reach, the ſame way from Tangent 45 to Tangent 52° 50′, as before.

With the Slider, for the ſecond.

This Proportion is direct; therefore ſet 77.5 on the Slide, againſt Radius on the fixt; then in a Line with 58.8 on the Slide you have 37.10 on the fixt Tangents, for the Comp. of the Courſe.

With Compaſſes, for the ſecond.

The Extent here is the very ſame as before, only that this Proportion being direct, the Tangent of the fourth Term is leſs than Radius, and therefore it muſt be reckoned forwards; hence it will be found to be 37° 10′, as before.

Having now determined the Courſe, the Diſtance may be found ſix ſeveral ways, that is, two Proportions may be raiſed upon each Side being made Radius, all of which ſhall be given for the ſake of Practice, and firſt by making the Diſtance to be Radius.

Variety 1. As S. Co. Courſe ·· Diff. Lat. : : Radius ·· Diſtance.
S. 37° 10′ ·· 58.8 : : S. 90 ·· 97 Miles.
2. S. Courſe ·· Departure : : Radius ·· Diſtance.

Second, By making the Diff. Lat. Radius.

3. As Radius ·· Diff. Lat. : : Sec. Courſe ·· Diſtance.
4. Tan. Co. ·· Departure : : Sec. Courſe ·· Diſtance.

Third, By making the Departure Radius.

5. As Tan. Co. Courſe ·· Diff. Lat. : : Sec. Co Cour. ·· Diſt.
6. Radius ·· Departure : : Sec. Co. Cour. ·· Diſt.

The fourth and fifth Proportions contain the greateſt Variety or Difficulty (if ſuch a Term may be uſed) as they involve both Tangents and Secants, ſome Obſervations therefore ſhall be made relative thereto, on the Scales, and the reſt remain for Practice.

The fourth, As T. Courſe ·· Depar. : : Sec. Courſe ·· Diſtance.
T. 52° 50′ ·· 77.5 : : Sec. 52° 50′ ·· 97.

Here the firſt Term being a Tangent and above 45°, it is plain that both the Tangent and Secant muſt be reckoned backwards from Radius, encreaſing towards the Left-hand, and as a Secant is always greater than any Tangent of the ſame Number of Degrees, and as a Secant is the third Term here, the fourth Term will be greater than the Second, or the Diſtance greater than the Departure, which is indeed a peculiar Property of the Hypothenuſe.

By the Slider.

In working, this Proportion muſt be reverſed, therefore fix the Braſs Index to Tangent of 52° 50′ on the fixt Piece, reckoning the ſame backwards from Radius, then bring Secant 52° 50′ on the Line of Sines (reckoning alſo backwards from Radius, on the ſliding Piece) to the Edge of the ſaid Index, which may be ſecured in that Poſition by turning the upright Skrew; move the Index to 77.5 Num. on the ſliding or middle Piece, and you will have by the Edge thereof, 97 Numbers on the lower or fixt Piece, for the Diſtance required.

The fifth, As T. Com. Cou. ·· Diff. Lat. : : Sec. Co. Cou. ·· Diſt.
T. 37° 10′ ·· 58.8 : : Sec. 37° 10′ ·· 97.

Here the firſt Term Tang. is leſs than 45°, and muſt be reckoned forwards, the third Term Secant contains the ſame Number of Degrees, but is greater than Radius, and muſt be reckoned backwards on the Sines, therefore the fourth will be greater than ſecond by juſt as much as the Secant exceeds the Tangent, to obtain which, obſerve the following Directions on the ſliding Gunter, viz.

Set Sec. 37° 10′, reckoned backward from Radius on the middle Piece, or Slider, againſt Radius on the fixed Piece, then will Radius on the Slider properly repreſent the Secant of 37° 10′ beyond the End of the Scale, the Diſtance of which Point from Tangent 37° 10′ will give the ſecond Term encreaſed to the fourth, the Slider being then in this Poſition, lay the Braſs Index on 37° 10′ on the lower fixed Tangents, and note what Degree and Minute on the ſliding Sines or Tangents, or any other Point thereupon, ſtands even with, or coincides with the ſame, which in this Caſe is at 37° nearly on the ſliding Sines; bring, or ſlide this Point (Sine 37) forwards to the fixt Radius, and then right againſt 58.8 Num. on the Slide, you will find 97 the Diſtance, on the fixt Num.

With Compaſſes.

Extend from Secant 0°, or S.90 to 37° 10′ reckoning backwards for Secants, lay that Extent from Tangent 37° 10′ towards the left, there reſt the Compaſſes; the Extent from that Point to T. 45° will reach from 58.8 Num. to 97 Num. as before.

Theſe Directions, together with what has been ſaid before, will, I hope, be ſufficient to explain not only the Manner, but the Reaſon of thoſe Kinds of Operations, and enable the Reader, with a little Practice, to diſpoſe the Terms, and work any Proportion with Eaſe, Diſpatch, and Propriety.

The laſt Thing in this Problem required, is the Difference of Longitude, which ſhall be found by the Sine Compliment of the middle Latitude; commonly called Middle Latitude Sailing.

PROBLEM XIV. continued.

By reſolving the Traverſe, the general Difference of Latitude is found to be 58.8 Miles, the Departure 77.5 Miles, Courſe 52° 50′ S. Weſterly, and Diſtance 97 Miles; What Latitude is ſhe in, what is her Diff. of Longitude, according to Middle Latitude Sailing, and Longitude in?

Latitude ſailed from 40° 00′ N.———40° 00′ N.
Diff. of Lat. = 58.8 m. or 0 59 S.

Latitude the Ship is in = 39 01 N.———39 01 N.

Sum = 79 01

Middle Lat. = 39 30
90 00

Comp. Mid. Lat. = 50 30

With this S. Co. of the Mid. Lat. and any two of the Terms given in the Queſtion, the Diff. of Longitude may be found; all the ſeveral Varieties of which, ſhall be given here, for the ſake of Practice — And,

First, By the Difference of Latitude and Courſe.

As,	S. Co.Mid.Lat. ..	Tan. Courſe	: :	Diff. Lat. ..	Diff. Long.
	S. 50° 30′ ..	T. 52° 50′	: :	58.8 m. ..	100 m.

Second, By the Departure.

As,	S. Co. Mid.Lat. ..	Radius	: :	Depart. ..	Diff. Long.
	S. 50° 30′ ..	S. 90	: :	77.5 m. ..	100 m.

Third, By Courſe and Diſtance.

As,	S. Co. Mid.Lat. ..	Diſtance	: :	S. Courſe ..	Diff. Long.
	S. 50° 30′ ..	97 m.	: :	S. 52° 50′ ..	100 m.

Or, By the following more unuſual Proportions.

Fourth, By the Departure only.

As,	T. Co.Mid.Lat. ..	Depar.	: :	Sec. Co. Mid.Lat. ..	Diff.Lon.
	T. 50° 30′ ..	77.5m.	: :	Sec. 50° 30′ ..	100 m.

Fifth, By Departure only.

As,	Radius ..	Depar.	: :	Sec. Mid. Lat. ..	Diff.Lon.
	Secant 0 ..	77.5m.	: :	Sec. 39° 30′ ..	100 m.

In the firſt Variety or Proportion, the ſecond Term Tangent is greater than the firſt Term Sine, or even than Radius, as it is more than 45°, therefore the fourth will be greater than the third, by as much as is that Diſtance, or Exceſs of the Tangent 52° 50′ beyond or above Radius, or the Sine 50° 30′ under, or ſhort of Radius; the Proportion then may be performed upon the Scales, in Manner following, viz.

By the Slider.

Set Sine 50° 30′ on the fixt Piece, againſt Radius on the Slide, then againſt T. 52° 50′ on the lower fixt Piece, you will have about 59° 45′ on the Tangent Slide; then draw out the Slide, and ſet the ſaid 59° 45′ Tangent againſt Radius, and in the right Line of the Index over 58.8 Num. you will find 100 Num. and with a little Attention it may be eaſily ſeen that the ſliding Radius has been removed forwards from Sine 50° 30′ to the fixt Radius, and by a ſecond Motion beyond the fixt Radius to 52° 50′ on the Tangents, at the ſame Time carrying 58.8 on the Line of Numbers along with it, to 100, the fourth Term required.

With Compaſſes.

The Extent from Tangent 45° to T. 52° 50′, being laid on the Sines from 50° 30′ backwards (which may be called the Reduction of Tangents to Sines) will reach to about 35° 30; then the Extent from this Point, or S. 35° 30′ to Sine 90, being laid from 58.8 m. on the Line of Numbers encreaſing, will reach to 100 m. *paulo plus*. The reſt are left for the Reader's Exerciſe.

Longitude ſailed from	=	5° 14′	W.
Diff. of Longitude 100 m. or		1 40	Weſterly.
Add. gives Long. the Ship is in	=	6 54	W.

The

The Uſe of the Scales in Mercator's *Sailing.*

This is very improperly called *Mercator's*, as it was the ſole Invention of Mr. *Edward Wright*, a Native of England, matriculated in Caius College, Cambridge, and flouriſhed about the Year 1590. Current Report has indeed cenſured *Mercator* for fraudulently, or in breach of Truſt, obtaining the Principles from *Wright*, but neither is this true, for *Gerrard Mercator* lived, and publiſhed his Univerſal Map of the World ſome Years before *Wright's* Correction of Errors appeared, nor was the laſt a Plagiary upon the firſt; but what ſeems to have given riſe to this erroneous Tradition, was this, viz. Mr. *Wright*, in Confidence, and under Promiſe of Non-publication, communicated his new inveſtigated Principles to one *Jodocus Hondius*, an Engraver of Flanders (but then in England) *Hondius* conſidered the Emolument or public Credit that would ariſe, as much ſuperior to a private breach of Promiſe, and publiſhed the ſame in Flanders, under his own Name; *Wright* called him to Account for this, he pleaded guilty, and made a poor ſhuffling Excuſe: This is the Fact, as it appears in Mr. *Wright's* Preface to his Book of Correction of Errors before-mentiond, publiſhed by Moxon.

It has been obſerved before, that Plane Sailing, as it is founded upon the Suppoſition that the Earth and Sea form one flat Surface, is erroneous; and this was not unknown to Navigators and Hydrographers many Centuries ago, (who well knew that the Meridians approached each other towards the Poles, and that this Decreaſe was in Proportion, as Radius, to the S. Co. of the Latitude) as appears by ſeveral Methods invented formerly, to accommodate this *Plane* to a *Spherical* Surface, ſo, as to be eaſy, familiar, and practicable at Sea, and demonſtrably correſpondent with Truth: but this Taſk was left (to the Honor of this Nation) for an Engliſhman to compleat — for Mr. *Edward Wright*, whoſe Name ought to be revered by every true Mariner, of what Kingdom ſoever he be.

As this is not intended to be a Treatiſe on Navigation, a full Explication of Mr. *Wright's* Principles and Superſtructure cannot be given here, I ſhall therefore beg Leave to refer thoſe who may be more curious in this Matter, to his Book, now mentioned; however, ſomewhat muſt be ſaid in order to explain the Baſis on which the Meridional Line on the Scale is founded. Mr. *Wright*, then happily and ingeniouſly hit upon the following Expedient. — Suppoſe a Spheric Superficies with Meridians, Parallels, Rhombs, &c. thereupon, be put into a concave Cylinder, their Axes agreeing in one: Let this Spheric Superficies ſwell like a Bladder (while it is blowing) equally always in every Part, 'till it apply and impinge itſelf upon the concave Superficies of the Cylinder; each Parallel encreaſing ſucceſſively, from the Equinoctial, until it comes to be of equal Diameter with the Cylinder, and conſequently the Meridians ſtill widening, until they come to be ſo far diſtant every where from each other, as they are at the Equinoctial; the Poles at laſt being ſuppoſed to burſt open, and the Cylinder ſlit and expanded, you have the *Modulus* of a Chart upon this Parallelogram, whereon the Situation of Places is in no wiſe diſtorted, the Meridians are become ſtreight Lines, and dilated, and the Diſtance of the Parallels enlarged at every Point of Latitude in the ſame Proportion, according to the Laws of Compound Motion: Now what this Proportion is, will demonſtrably appear in the Diagram, (*Plate* 1. *Fig.* 4.) wherein we will ſuppoſe the Latitude to be 60 Degrees, the Complement whereof is 30 Degrees: Deſcribe the Semi-circle F I P B, as a Meridian; draw the Line F A B to repreſent the Equator, alſo quarter it with the Meridian P A; then lay off the Latitude, 60 Degrees, from B to E, whoſe Compliment is E P, equal to 30 Degrees; draw the chord Line E I, the half of which, E K, is the Sine of the Arc E P, the Compliment of the Latitude; erect the Perpendicular B C, which will be a Tangent to the Semi-circle, and draw the Line A C through E, until it meets the Tangent in C; then will A C be the Secant of the Latitude 60°, or the Arc B E; laſtly, draw E D Parallel to A P, which will be the Sine of the Arc E B, equal to 60 Deg. the Latitude.

Now if we conceive this Semi-circle to be half a Bladder Globe, and to be dilated by an equal uniform *Flatus*, the Point E (as the Extremity of the Radius A E) in Lat. 60°, will be affected (as it were) by a compound Force, the one urging it towards the North to M, the other at the ſame Time urging it towards N, but being acted upon by both Forces united, one at right Angles to the other, the Point E is obliged to take its Courſe in the Diagonal of the Parallelogram E M C N, and will impinge the Cylinder in C. Now this Diagonal E C being created by the Motion of the Point E, from the Extremity of the Radius A E, forms the whole continued Line A C, which is evidently the natural Secant of the Arc E B, or the Latitude: The Radius then (under this regular *Flatus*, or Diſtenſion) is encreaſed to a Secant, and determines the proper Length of a Degree of Latitude in the Parallel of 60° in Proportion to the Radius, while at the ſame Time the Meridians are in Motion, and uniformly increaſing and approaching to right Lines, which will take Place as ſoon as every Point therein has impinged the concave Cylinder, when they will become Parallel to each other, for then the Radii of the leſſer Circles, or Sine Comp. of the Latitudes, will be increaſed, dilated, or become equal to the Equatorial Radius, or whole Sine: The Rhombs will become right Lines alſo, and interſect the Meridians at equal Angles, which is their peculiar Property.

But although it has been above obſerved, that a Rectangular-compound-motion acting upon the Point E (as the Extremity of the Radius) will direct it by the Diagonal to C, where it will impinge, and compleat the Secant, and conſequently be limited by a Tangent upon the Cylinder, yet, in this Conſtruction, theſe Tangents ſerve only as *Agents* (if I may be allowed the Expreſſion) in order to limit the Triangles, and thereby explain the Theory.

For, at the ſame Time it is to be attentively regarded, that Mr. *Wright's* Bladder Meridians, although they preſerve this *Ratio*, yet they do not take this Route, or Courſe in ſwelling, nor is it poſſible that they ſhould, for all the Meridians ſtand at right Angles to the Equator, and muſt increaſe in that Direction; but the Point E, conſidered as the Extremity of the Radius A E, moves obliquely to the Equator in the Diagonal E C, in order to compleat the Increment of the Radius, which then becomes the proper and determinate Secant at C, and this will be univerſally the Caſe, let the Point E be taken in the Meridian where it may; yet the ſuppoſed Motion of the Radius, and the ſwelling or increaſed Length of the Meridians (ſtanding at the right Angles) by the Action of the uniform *Flatus* are both in the ſame *Ratio*, and this Diagonal will conſtantly be the Increaſe of Radius in ſuch Proportion as Radius has obtained upon the Sine Compliment.

To form diſtinct Ideas, and thence an adequate and true Conception of theſe Principles, requires the Reader's Care and ſtrict Attention, leſt he ſhould be betrayed into the fallacious Opinion of a Tangential Property in this Conſtruction, which has been the Caſe with *ſome* (though in other Reſpects able Writers) who ſeem to have miſtaken Mr. *Wright* in this Point, and have gone ſo far as to pronounce *this Chart* to be erroneous; and that, if any Thing, it ought to poſſeſs a Tangential Property, and not a Secantial; which Miſconception ſeems to have ſolely proceeded from their ſubſtitution of the *Modulus*, or Cylinder, when compleated with the Secant and Tangent Lines, ſlit open and extended to a plane Paralellogram, for the real and proper Chart, which is not the Caſe, nor can it be admitted as a competent and practical Mariner's Chart, ſince the Rhombs under this Tangential Projection, will not be right Lines. Vid. Phil. Tranſ. Vol. liii. for 1763.

In order then to remove this Ambiguity, and convey ſome ſome further Ideas concerning Mr. *Wright's Flatus*, and the Manner in which the Meridians are extended, let us advert to *Fig.* 1. *Plate* 1. where F A B repreſents the Equator; P E B a Meridian; B E an Arc of it equal to 60 Degrees: And then conſider the Motion of the ſaid Arc during the *Flatus*; under which we ſhall find the ſaid Meridian, and Arc ſtreightening towards B C, as the Semi-diameters of the Parallels increaſe, and at the ſame Time lengthening towards the Pole in the ſame *Ratio*, the Point E at the End of the Arc moving therewith through the Space E S, until the Arc becomes ſtreight, which will be ſuch, when the Co. Sine K E becomes equal to Radius, and then at the ſame Time the ſaid Arc will be lengthened to the Secant, and the Point E therein will coincide with S; for B S then becomes equal to A C the proper Secant.

Again, if further Illuſtration may be required; let it be conceived that the Meridian A P (equal to A E) which ſtands at right Angles to the Equator F B, will, by the *Flatus* be uniformly increaſed to T, upon the Concavity of the Cylinder, while A E is continued, or extended to C, and then P T will become equal to E C, equal to the Increment in either Line, and this holds good with reſpect to every Meridian, as they are all at right Angles to the Equator upon the inſcribed Globe, and increaſe Secantially in that Poſition (as from a Center) towards ſtreight Lines, until they adhere to the Cylinder: Hence we are enabled Geometrically to demonſtrate the *Ratio* of the Increments, and from thence Trigonometrically compute the ſame, for the Conſtruction of a true and practical Sea Chart; and from hence it will appear, that Secants being drawn through every Minute of Latitude, and transfered to the Meridian A T, will conſtitute a Line of natural Secants, and ſhew the increaſed Length of each Minute reſpectively upon the Meridian, in Proportion to the Length of the Radius or of one Minute at the Equator.

In this Diagram then, there are two right angled Triangles, viz. A B C and A D E. which Triangles are ſimilar, therefore their Homologous Sides are in Proportion, that is,

A D ·· A B : : A E ·· A C.
Sine Co. Lat. ·· Radius : : Radius ·· Secant of ſame Lat.

Or in Words more at large; ſuch Proportion, directly, as the Sine Comp. Lat. or Semi-diameter of the Parallel, hath to Radius, or Semi-diameter of the Equator; the ſame hath the Radius, or whole Sine, to the Hypothenuſe or Secant of the ſame Latitude: Hence it is evident that, as the Line AD=K E is diſtended, dilated, or encreaſed to the Length of A B=K N, ſo is the Line A E increaſed to the Length A C; and every Point of Lat. on the Meridian will thus move towards the Poles in a Secantial Ratio, while the Meridians themſelves are ſeparating, widening, and diſtending until they become ſtreight Lines and Parallel to each other; the Rhombs alſo (every Point therein) moving or flowing at the ſame Time in, and with the Meridians, will neceſſarily terminate in ſtreight Lines, and thence continue to retain their known Property to interſect the Meridians at equal Angles. The Meridians are then to be conſidered, or conceived to be as in a continual flowing State; hence it is neceſſary, to obtain the Value of the Secantial Increments to very ſmall Arches, or Portions of Latitude, not exceeding one Minute, (or even if they did not exceed Halves or Quarters it would yet be more accurate) beginning from the Equator; and theſe being found and added ſucceſſively to the former, will give the proper Secant in natural Numbers for the Section or Equatorial Diſtance of every Degree, Minute, &c. of the Parallel of Latitude to be laid off from any Scale of equal Parts, upon a Nautical Chart, which Numbers form a Series of Secants, and when compleated and arranged in order, are called by the Name of Tables of Meridional Parts, in all the more modern Books of Navigation, and give Foundation to theſe two Lines upon the Scale, which may be there laid down with a *proper* Equatorial Line annext; I call it a *proper* Line, becauſe the Size or Length of an Equatorial Degree muſt be the ſame in both.

Theſe Principles duly conſidered, it will not be difficult to conceive that the Line B S, equal to the Secant A C, will not abſolutely repreſent the Equatorial Diſtance, or determine the

Point of the Parallel-Section in the Meridian, at the Latitude of 60 Degrees, but only the Length of an Arc of 1 Minute in that Latitude, that is, of an Arc intercepted between the Parallels of 59 Degrees, 59 Minutes, and 60 Degrees, in Natural Numbers.

For if we ſuppoſe (as is truly the Caſe) the Radius A B of the preceeding Sphere, or Elementary Globe, to be equal to 1 Minute of a Degree, and an Equatorial Minute, or Minute of Longitude, to be equal to Unity alſo, conſidered as a Natural Number, with any Number of Cyphers annext, then will the Length of the ſeveral and reſpective Secants obtained by the foregoing Trigonometric Proportion, in Natural Numbers, be the true Length of 1 Minute at the ſeveral reſpective Latitudes on the Nautic Meridian, in Proportion to an Equatorial Minute of any aſſigned Magnitude.

Hence the Meridian Line on any Chart will be conſtructed by laying, or projecting thereon the Secants thus reduced to Natural Numbers, to every Minute of a Sphere or Globe, whoſe Radius is ſuppoſed to be equal to Unity, or 1 Longitudinal Minute of any aſſigned Magnitude, ſuited or adapted to the Size of the intended Chart.

And in order to obtain theſe Increments, or the correſpondent Length of any ſmall Arc, upon the encreaſed or dilated Meridian, let us ſuppoſe a Minute of the Equator to be divided into, or to contain 10,000,000 equal Parts, and the firſt Minute from the Equator upon the diſtended Meridian to contain 10,000,000 equal Parts alſo (in manner as Mr. *Wright* has done) then will this Length point out in the ſaid Meridian, where the Section of one Minute of Latitude is to paſs, extreamly near the Truth, though not abſolutely ſo, for the ſeveral Seconds in this firſt Minute will be intitled to ſome very ſmall Increment, but this will be rather more than ſufficiently compenſated when we come to add the Increments of the ſucceſſive Minutes, as they give the Equatorial Diſtance, or Section of the ſuperior Minute. Then for the Parallel of two Minutes, that is, the Increment between one and two; and herein (as well as in all the reſt) we ſhall have Occaſion for a Proportion deduced from the latter Part of the foregoing, where it has been proved, that the Increment of the Meridians is, as Radius is to the Secant of that Arc; therefore the following Proportion will determine the Extent, or Breadth of each reſpective Minute of Latitude upon the dilated Meridian, viz.

As Radius ·· Secant of 2 Minutes : : the natural Number 10,000,000, the Diviſional equal Parts in one Minute, ·· 10,000,002, the equal Parts contained between the firſt and ſecond Minute of Latitude, and this added to 10,000,000 the Breadth of the firſt Minute, gives 20,000,002 for the Diſtance of the ſecond Minute of Latitude from the Equator; again for 3 Minutes, it is,

As Radius ·· Secant of 3 Minutes :: 10,000,000 ·· 10,000,004 the encreaſed Breadth from the ſecond Minute to the third, or the Increment preciſely at the third or ſuperior Minute always, and this added to the preceding Sum 20,000,002, gives the Section or Diſtance (30,000,006) of the third Minute from the Equator; in like Manner proceed for the fourth, fifth, ſixth, and all the reſt as high as 89° 59′ of Latitude; but this Work may be ſomewhat curtailed, by leaving out the three Figures next the Right-hand in all the Terms, which is ſufficiently accurate for every practical Purpoſe; and if a greater Degree of Accuracy be required, theſe Secantial Proportions may be repeated and computed to every 10 or 15 Seconds, for then theſe Parallel Diſtances would be leſs deſultory, and approach nearer to the true Increments of the the *Flatus*, or Secantial fluent State, and therefore to vie with Perfection itſelf; Mr. *Oughtred*, Sir *Jonas Moore*, Dr. *Wallis*, Dr. *Halley*, and others, have been induced to find Methods of conſtructing theſe Tables with more Accuracy than by the Addition of Secants, as here explained; but the moſt curious of theſe Methods only ſerves to prove that Mr. *Wright's* Tables do no where exceed the true Meridional Parts by half a Minute, and this only near the Poles, for in Latitudes, as far as Navigation is practicable, the Difference is ſcarce ſenſible: See Mr. *Robertſon's* Elements of Navigation on Globular Sailing.

The intelligent Reader will preſently diſcover from what has been ſaid, that the ſeveral ſucceſſive Increments, or Meridional Lengths of each Minute, or half Minute, &c. if ſo computed, being arranged and tabulated in Order from the Equator, will form a Sett of Natural Secants, and may be uſed as ſuch; but when added ſucceſſively to each other, their reſpective Sums at every Minute, &c. conſtitute a Series of Natural Secants, and denominate the Table of Meridional Parts; but theſe Tables in Nautical Books, ſeldom exceed four or five Places of Figures, being near enough the Truth for common Purpoſes. Hence it may be further obſerved, that from a good Table of Natural Secants (ſuch as *Sherwins)* a Table of Meridional Parts or Equatorial Diſtances to Minutes of Latitude may eaſily be obtained by the conſtant Addition of theſe Increments, or Natural Secants, to the Sum of thoſe preceding, beginning at the Equator, or 0 Deg.

More has been ſaid upon this Subject than was at firſt intended; but if any Difficulties are removed hereby, and our excellent Author's true meaning conveyed in a Manner leſs liable to Doubts or Objections, the Labour then will not be loſt.

PROBLEM XV.

Both Latitudes and Courſe given, to find the Diſtance and Difference of Longitude by the Meridional Parts, or *Wright's* Sailing, erroneouſly called *Mercator's*.

EXAMPLE.

Taken from Problem XIV. in order to compare and prove the Truth of the Reſults.

A Ship ſails S. 52° 50′ W. from a Port in 40 Degrees North Latitude; and the next Day, by Obſervation is found to be in Lat. 39° 01′ N. her Diſtance Run, and Difference of Longitude are required?

Lat. ſailed from	40° 00′N.	Merid. Parts	2623	from Tables
Lat. come to	39 01 N.		2546	
Diff. of Lat.	0 59 S.	Mer.Diff.Lat.	77	

By the Meridional Lines on the Gunter.

Beſides the Contrivance of the regulating or adjuſting Screws, the late truly meritorious Mr. JOHN ROBERTSON, has conſiderably improved this Scale, not only by encreaſing its Length, and thereby the Diviſions and Sub-diviſions of all the Lines thereupon with great Accuracy, but more eſpecially *this*, of the Meridional Parts, by breaking it into two Lines, the firſt beginning at the Equator, and extending to 50 Deg. of Latitude, the ſecond re-commences at 50 Deg. at the Right-hand, or Radius End of the Scale, and reacheth to 75 Deg. of Lat. nearly, beyond which the Meridional Parts are ſeldom wanted, as hath been obſerved before; by this Means the Diviſions are greatly augmented, and fitted to a Scale of equal

Parts of half an Inch to a Degree, which is ſufficiently large for the Conſtruction of moſt Nautical Charts, and for finding the Mer. Diff. of Lat. with more Accuracy in this Kind of Sailing, when Tables computed are not at Hand.

General Directions for finding the Meridional Difference of Latitude, with Compaſſes or without.

1. With the Compaſſes.

The Extent on the Meridian Line (with fine Points) from the Latitude ſailed from, to Latitude arrived at, meaſured on the Line of equal Parts annext, from the Beginning at the Right-hand, will give the Degrees and tenth Parts, which multiplied by 60, taking in the odd Minutes, gives the Meridional Difference of Latitude required.

2. With the Index.

Set the Edge of the Braſs Index exactly againſt the Latitude ſailed from, and *note*, the Degrees and tenth Parts in a Line with it on the Equatorial Scale; then move the Index to the Latitude come to, and *note* the Degrees and tenths right againſt that; ſubtract the leſs from the greater, and the Remainder is the Mer. Diff. of Lat. in Deg. and tenths, which multiply by 60, taking in the odd Minutes, gives as before.

Note, The Meridional Deg. and alſo the equal Part Deg. are ſub-divided into tenths, therefore the Value of every Subdiviſion is ſix Minutes.

EXAMPLE.

The Meridional Difference of Latitude correſpondent to the proper Difference of Latitude given in this Problem, is required?

1. With Compaſſes.

The Extent from Lat. 40 Deg. to 39° 01′, laid from the Beginning of the Line of equal Parts, at the Right-hand, will reach to 1·3 Deg. nearly, that is, 1 Deg. 18 m. or 78 min. or rather 77 min.

2. With

Plate 1.

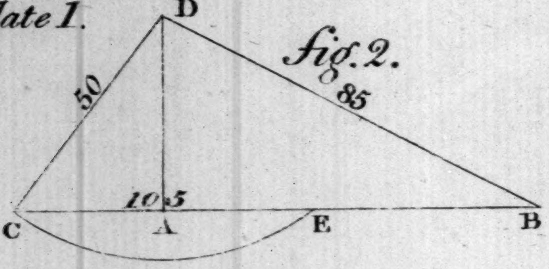
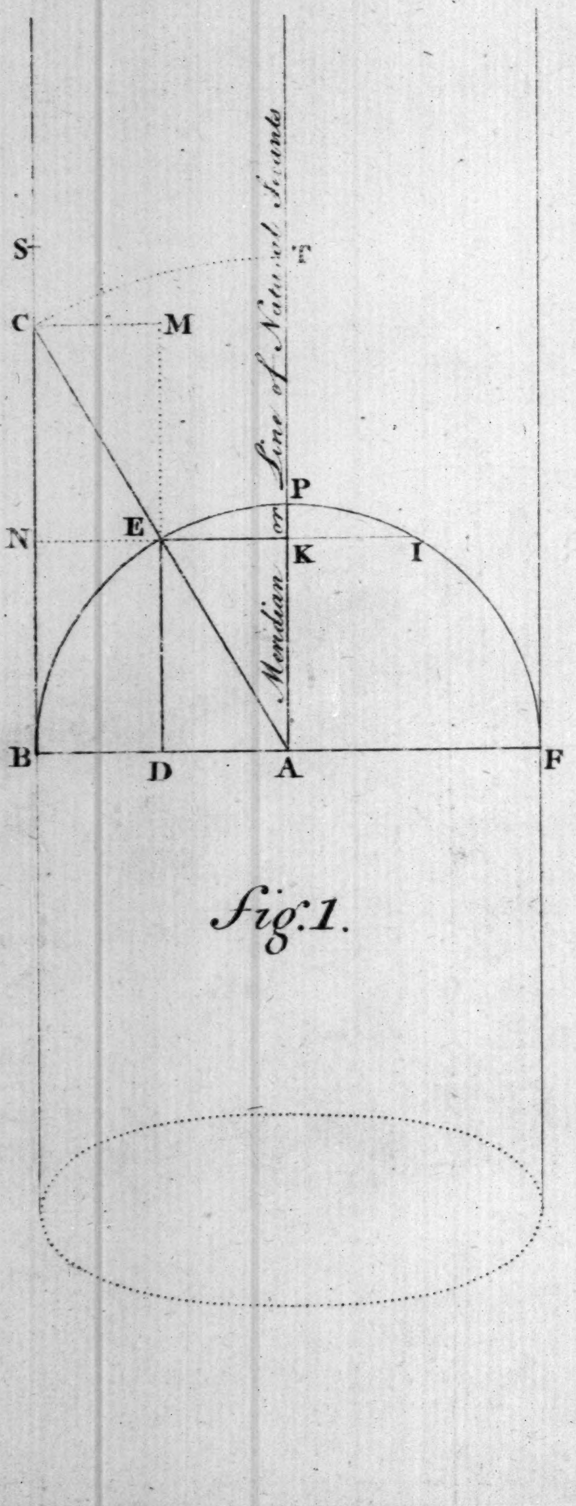


fig. 2.



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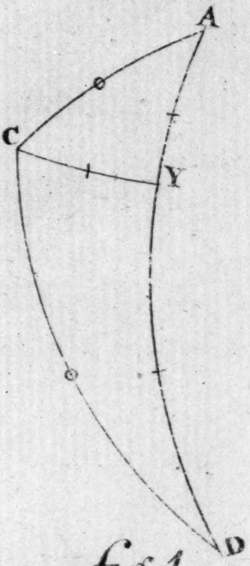

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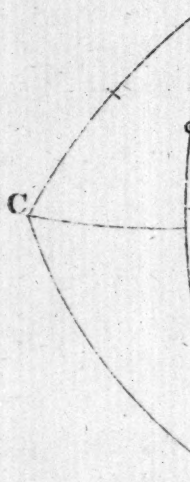

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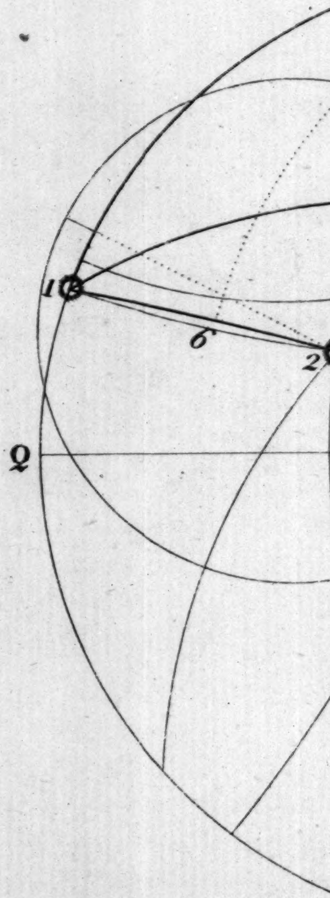

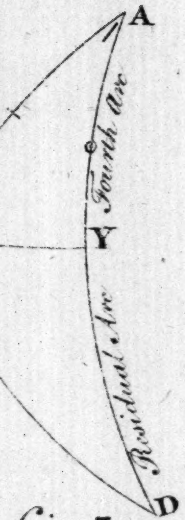

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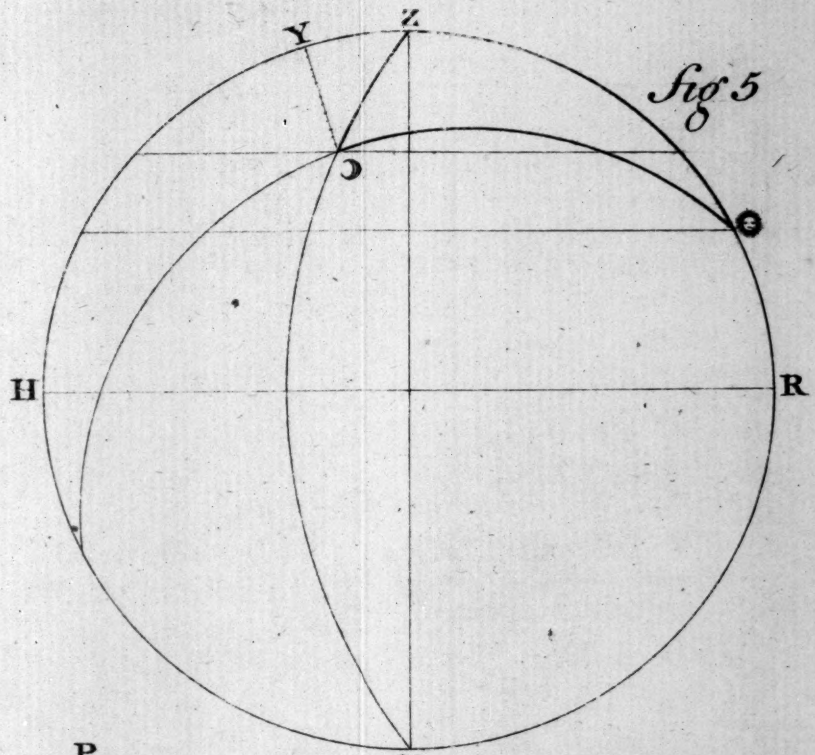


fig 5

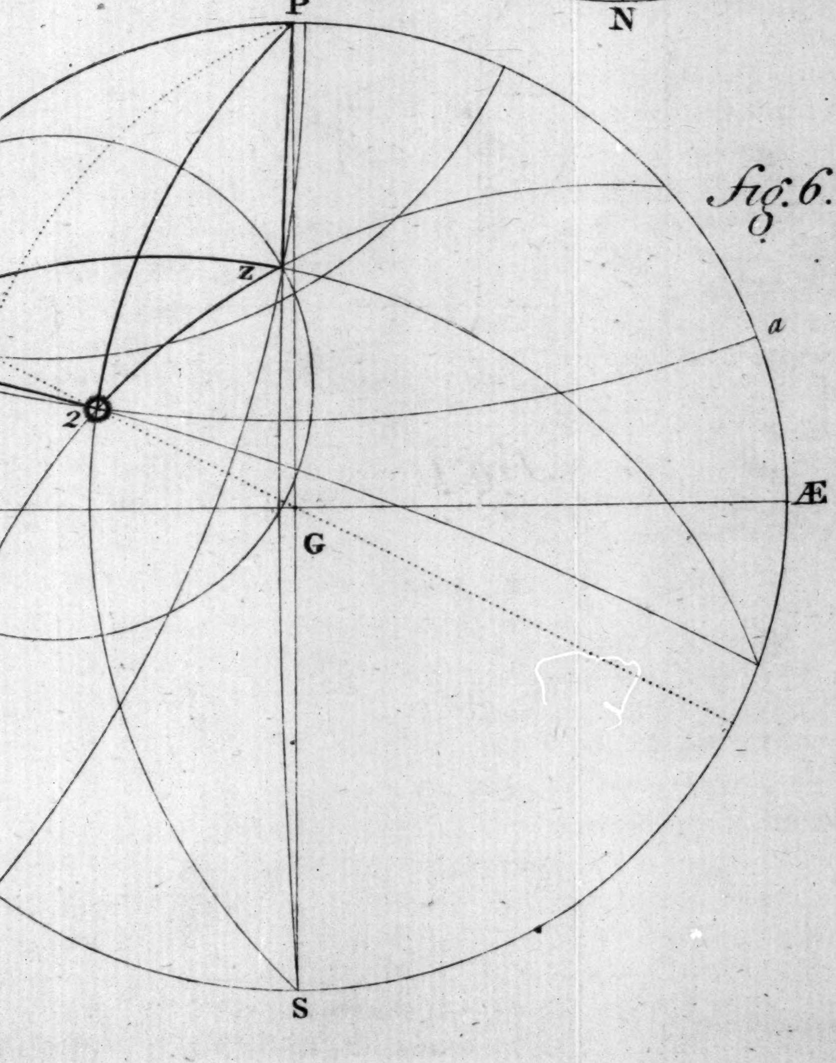


fig. 6.

2. With the Braſs Index.

Set the Edge thereof exactly to 40°, the Latitude ſailed from, on the Meridional Parts, againſt which, on the Equatorial Line, you will find 43.7 Deg. then remove the Index to 39° 01′, the Lat. arrived at, on the Mer. Line, and againſt that will be found 42.4 Deg. which ſubtract from the above (becauſe the Latitudes are both N.) leaves 1.3 Deg. as before.

The Meridional Difference of Latitude being thus found, the Proportions for the Difference of Longitude are as follow:

1. The Hypothenuſe, Radius.

As, S. Co. Courſe ·· Mer. Diff. Lat. : : S. Courſe ·· Diff. Long.
S. 37° 10′ ·· 77 m. : : S. 52° 50′ ·· 101.5 m.

2. Mer. Diff. Lat. Radius.

As, Radius ·· Mer. Diff. Lat. : : Tan. Courſe ·· Diff. Long.
Tan. 45° ·· 77 : : Tan. 52° 50′ ·· 101.5 m.

3. Diff. Long. Radius.

As, Tan. Co. Courſe ·· Mer. Diff. Lat. : : Radius ·· Diff. Long.
Tan. 37° 10′ ·· 77 : : T. 45° ·· 101.5 m.

Which exceeds the ſame determined by middle Latitude, by about one Mile and an half.

PROBLEM XVI.

The Latitude and Longitude of two Places given, to find the Courſe and Diſtance.

EXAMPLE.

What is the Courſe and Diſtance from the Lizard to the Iſland of Barbadoes?

	Lat.	Mer. Parts	Lon.	
Lizard	49° 57′N.	3470	5° 14′W.	By Tables.
I. Barba.	12 58 N.	785	58 50 W.	
Proper Diff.Lat.	36 59 S.	M.D.La. 2685	D.Lon. 53 36.	
	60		60	
	Or, 2219 Min.		3216 Min.	

By the Meridional Line on the Scale with the ſliding Index.

Set the Index to the greater Lat. 49° 57′ on Mer. Line, and by the Edge, on the equal Parts, you will have 57° .85, then remove the ſaid Index to 12° 58′, the leſs Lat. on Mer. Line, and againſt that ſtands 13° .1 on the Equatorial Line; ſubtract the leſs from the greater, there remains 44° .75 or 44° 45′, which multiplied by 60, gives 2685 min. the Mer. Diff. Lat. required, as before.

By the Compaſſes.

The Diſtance between the two Lat. on Mer. Line, laid upon the equal Parts from the Right-hand, will reach to 44° 45′, or 2685 as before. If the Extent be too great for the Compaſſes, take one half between the Latitudes, lay that upon the equal Parts, and the double of what you here obtain is the Mer. Diff. of Lat. required.

PROBLEM XVI. CASE II.

Both Latitudes given, but one of them more than 50 Degrees, the other leſs; in this Caſe to find the Meridional Difference of Latitude by the Scale.

EXAMPLE.

Two Places are ſituated, the one of them in Lat. 60° 30′ N. the other in 30° 30′ N. what is the Meridional Difference of Latitude between them?

Lat. of the firſt Place	60° 30′ N.	Mer. Parts	4588	by Tables.
Lat. of the ſecond	30 30 N.		1923	
Proper Diff. of Lat.	30 00	Mer. Diff. Lat.	2665	

It muſt be obſerved, that in order to render the Mer. Parts on this new Scale more accurate and uſeful, the uſual Line upon former Scales is here broke into two; the firſt extending to Lat 50 Deg. on the Left, and the ſecond recommencing with 50 Deg. at the Right-hand, is continued to 75 Degrees; both theſe Lines are ſubject to one Equatorial Line, and therefore, when one given Lat. is above 50 Deg. and the other below; to find the Mer. Diff. Lat. by the Scale, a little more Trouble is required, and the ſame is performed thus, ſet the Braſs Index to the leaſt Latitude, and note down the Equatorial Parts anſwering thereto, ſubtract theſe from 58 (the whole Number of primary equal Parts on the ſaid Line) the Remainder will be the Value of the equal Parts, correſponding with the Mer. Parts contained between the leaſt Lat. and 50 Deg. at the End of the Scale; then move the Index to the greater Latitude, and note the equal Parts anſwering thereto, reckoning from the Right-hand; this Number added to the Remainder, gives the Meridional Difference of Latitude

The ſame with Compaſſes.

Extend from the leſſer Latitude to 50°, at the End of the Scale, and note what this makes on the Equatorial Line; then take off the equal Parts correſponding with the greater Lat. the Sum of theſe two is the Mer. Diff. Latitude required.

In the preſent Example with the Index.

Set the Index upon 30° 30′, againſt which you have on equal Parts	32.1 nearly.
Subtract this from the whole Line	58.0
Remainder	25.9
Then ſet the Index upon 60° 30′; anſwering thereto is	18.5
Added, gives the Meridional Diff. Latitude =	44.4

Which, as every tenth is 6 min. makes 44 Deg. 24 min. and multiplied by 60, gives 2664, being only an Uuit leſs than thoſe found by the Tables.

Note, If the Line of equal Parts be ſub-marked with the Numbers, 1, 2, 3, &c. from the Left-hand towards the Right, you will obtain the Complements to 58 Deg. without the Trouble of Subtraction as above; for, as the primary Diviſions are all equal, ſo are the Secondary or Subdiviſions, and may indifferently (as the Caſe requires) be reckoned either way.

The ſame with Compaſſes.

The Extent from 30° 30′ to 50° at the End, laid upon the Equatorial Line from 0, gives 25°.9; then the Extent from 50° at the Beginning of the ſecond Line, to 60° 30′, gives 18.5, which added to the former, makes 44°.4 as before.

There remains one Variety more, for the Scale Mer. Parts, with which I ſhall finiſhthis Kind of Sailing.

PROBLEM XVII.

One Latitude, Courſe and Difference of Longitude given; to find the Diſtance and Latitude arrived at.

EXAMPLE.

A Ship ſails S.W. by W. from the Lizard, until ſhe be in Long. 57° 26′ W. What Lat. is ſhe in, and her Diſtance ſailed?

Long. ſailed from 5° 14′ W.
Long. the Ship is in 57 26 W.

Sub. gives Diff. Long. 52 12 W.
60

Or 3132 min.

To find the Mer. Diff. Lat. any of the Sides may be made the Radius; and firſt, the Hypothenuſe: Then,

As, S. Courſe .. Diff. Long. : : S. C. Courſe .. Mer. Diff. Lat.
S. 56° 15′ .. 3132 : : S. 33° 45′ .. 2093 Min.

Second, Mer. Diff. Lat. Radins.

As, Tan: Courſe .. Diff. Long. : : Radius .. Mer. Diff. Lat.
Tan. 56° 15′ .. 3132 M. : : Tan. 45° .. 2093 Min.

Third, Diff. Long. Radius.

As, Radius .. Diff. Long. : : Tan. Co. Courſe .. Mer. Diff. Lat.
T. 45° .. 3132 : : Tan. 33° 45′ .. 2093 Min.

By the Scale the equal Parts anſwering to Lat, 49° 57′ are 57.85, or 57° 51′, } or = 3471 Mer. Parts.

Sub. gives the Mer. Parts for the Lat. the Ship is in 1378

Which ſeek for in the Table of Mer. Parts, and againſt it you will find 22° 22′, the Latitude the Ship is in, North; and to find this by the Scale, Obſerve the following Directions, viz. take the Meridional Diff. of Lat. being here 2093 m. or 34° 53′, from the Line of equal Parts, or if the Extent be too great for the Compaſſes (as in this Caſe it is) then take the one half, and ſet it from the Lat. departed from, here 49° 57′, on the Meridional Line decreaſing, becauſe it is N. Lat. ſailing S.ly, once, if the Parts be taken at one Extent, or twice, if you take but the one-half, and it will reach at twice (in this Caſe) on the ſame Line to 22.4 tenths, or 22° 24′ the Latitude the Ship is in, differing from the Lat. above found by the Table, only two Minutes.—The Diſtance is now eaſily found, if neceſſary.

PARALLEL SAILING.

PROBLEM XVIII.

Two Places in one Parallel of Latitude, their Difference of Longitude given; to find their Diſtance.

EXAMPLE.

The Diſtance between the Lizard and Pengwin Iſland near Newfoundland is required?

Pengwin Iſland	Longitude is	52°	50′ W.
Lizard ———		5	14 W.
Sub. remains Diff. Long.		47	36
		60	
		Or 2856 min.	

Both the Places are ſuppoſed to lie in Latitude 50 Deg. the Complement of which is 40 Deg.

By Similarity of Triangles, the Proportion will be,

As, Radius .. S.c.Lat. : : Diff. Long. .. Diſt. in par.
S. 90Deg. .. S. 40Deg. : : 2856 .. 1836 min.

By the Plane Scale.

Take the Latitude of 50° from the Scale of Chords, and meaſure it from 60 on the Line of Miles of Longitude thereto annext, marked M. Lon. this points out Minutes 38.6 for the Extent of a Degree of Longitude in that Parellel of Latitude; then ſay,

As 1 Deg. ·· 38.6m. ·· 47.6 Deg. ·· 1836 Miles, the Parallel Diſtance as before.

PROBLEM XIX.

Two Places in one Latitude, their Diſtance given; to find their Diff. of Longitude.

EXAMPLE.

If a Ship ſails Weſt 390 Miles from the Lizard, what Diff. of Long. has ſhe made?

Proportion. S. c. Lat. ·· Radius : : Diſtance ·· Diff. Long.
S. 40° 03′ ·· S.90 Deg. : : 390 miles ·· 606.1 miles.

By the Plane Scale.

Right againſt Lat. 50° on the Chords, you will find 38.6 m. on M. Lon. annext, for the Value of one Degree in that Parallel; then ſay,

As 38.6 Miles ·· 1 Deg. : : 390 m. ·· 10.1 or 606.

PROBLEM XX.

Two Places in one Parallel of Lat. their Diſt. and Diff. of Long. given to find that Parallel of Lat.

EXAMPLE.

Suppoſe two Ships upon the Equator 2842 Miles aſunder, and they both ſail North equal Diſtances, until they are but 1833 Miles aſunder, what Lat. are they in?

By the Plane Scale.

As 2842 m. ·· 1833 m. : : 60 m. ·· 38.6 m. for the Value of 1 Deg. of Longitude in the Parallel of Latitude required.

Then, from the Line of Long. marked M. Lon. take 38.6, which meaſure on the Line of Chords thereto annext, and you will find 49° 50′ for the Lat. the Ships are in.

PROBLEM XXI.

To project, or conſtruct a true Nautical Chart by the Help of the Lines of Meridional Parts, and equal Parts laid down upon the Gunter Scale.

Upon the old Gunters there was but one Merid. Line, of about 24 Inches in Length, the Diviſions and Sub-diviſions then were conſequently very ſmall, and unfit for Practical Uſe: This Line Mr. *Robertſon* has now formed into two; the firſt begins at the Equator, and extends to 50 Deg. of Latitude; the ſecond re-begins at 50 towards the Right-hand, and is continued to 75 Deg. nearly, and each of theſe running almoſt the Length of the Scale, or 30 Inches, the Diviſions are therefore enlarged to half an Inch for a Degree, which is ſufficiently capacious in moſt practical Caſes; but if a larger, or any other Scale be required, a Chart may be conſtructed ſuitable thereunto, with a very little additional Trouble, as ſhall be preſently ſhewn; in order then to give a full Explanation, it may be proper to give an Example under three Varieties, and firſt,

Variety 1. To conſtruct a *Wright's* Nautical Chart, from the Lines of Mer. Parts on the Scale, beginning at the Equator, and extending North or South, or both at Pleaſure, and containing any determined Number of Degrees of Longitude, Eaſt or Weſt, or both, from a fixt Meridian.

This is ſo familiar that there needs no exemplary Plan to explain it; a very few Directions will be ſufficient.

The Quality and Extent of the Chart being determined, draw a Line to repreſent the Equator, take the Number of the Degrees of Longitude intended, from the Equatorial Line upon the Scale, and lay them upon the Equator; this limits the Breadth of the Chart: Then erect Perpendiculars upon each Extremity of the Equator (taking great Care that this be accurately done) and upon theſe Perpendiculars (now Meridians) lay off the greateſt Extent of the Latitude in Degrees, either North or South, or both, (taken in your Compaſſes from the Line of Meridional Parts at the Beginning) from the Equator on the Chart, and join theſe two Northern or Southern Extremities by a Parallel of Latitude drawn between them; theſe four Lines will form a Parallelogram, and limit the true Size of your Chart, if the Angles be right, and the Parallels of equal Length: The interior Work may then be readily compleated; and firſt, the Degrees of Longitude being all equal, you may divide and ſub-divide the Equator equally into the Number of Degrees propoſed, or take them ſeparately and accurately from the Equatorial Line on the Scale, and lay the ſame on your Equator; in like Manner may the half Degrees and Quarters be laid off: And ſecondly, for the Degrees of Latitude; the Extent of each reſpective Degree from the Equator taken from the Beginning of the Meridional Line on the Scale, and laid on the two extream Meridians, from the Equator on your Chart, either Northward or Southward, or both, if your Chart be ſuch, will give the Points in the ſaid Meridians, where the reſpective Degrees, or Parallels of Latitude are to be drawn; in like Manner may the Halves and Quarters be laid off; it may be obſerved, by way of Checque, that the Length of the Degrees of Latitude are conſtantly increaſing towards the Poles.

If it was required to lay down the Sea Coaſts and Iſlands upon the Chart, this can only be done by the known Latitudes and Longitudes of the more capital Places, Headlands, Iſlands, &c. which may be taken from the modern Tables, or thoſe recently corrected, ſuch as Mr. *Robertſon's* Elements of Navigation, Mariners Compaſs rectified, Mariners Calendar, and ſome others.

Variety 2. To conſtruct a true Nautical Chart, beginning at any Degree removed from the Equator N. or S. and extending North or South at Pleaſure; with any Number of Degrees of Eaſt or Weſt Longitude aſſigned.

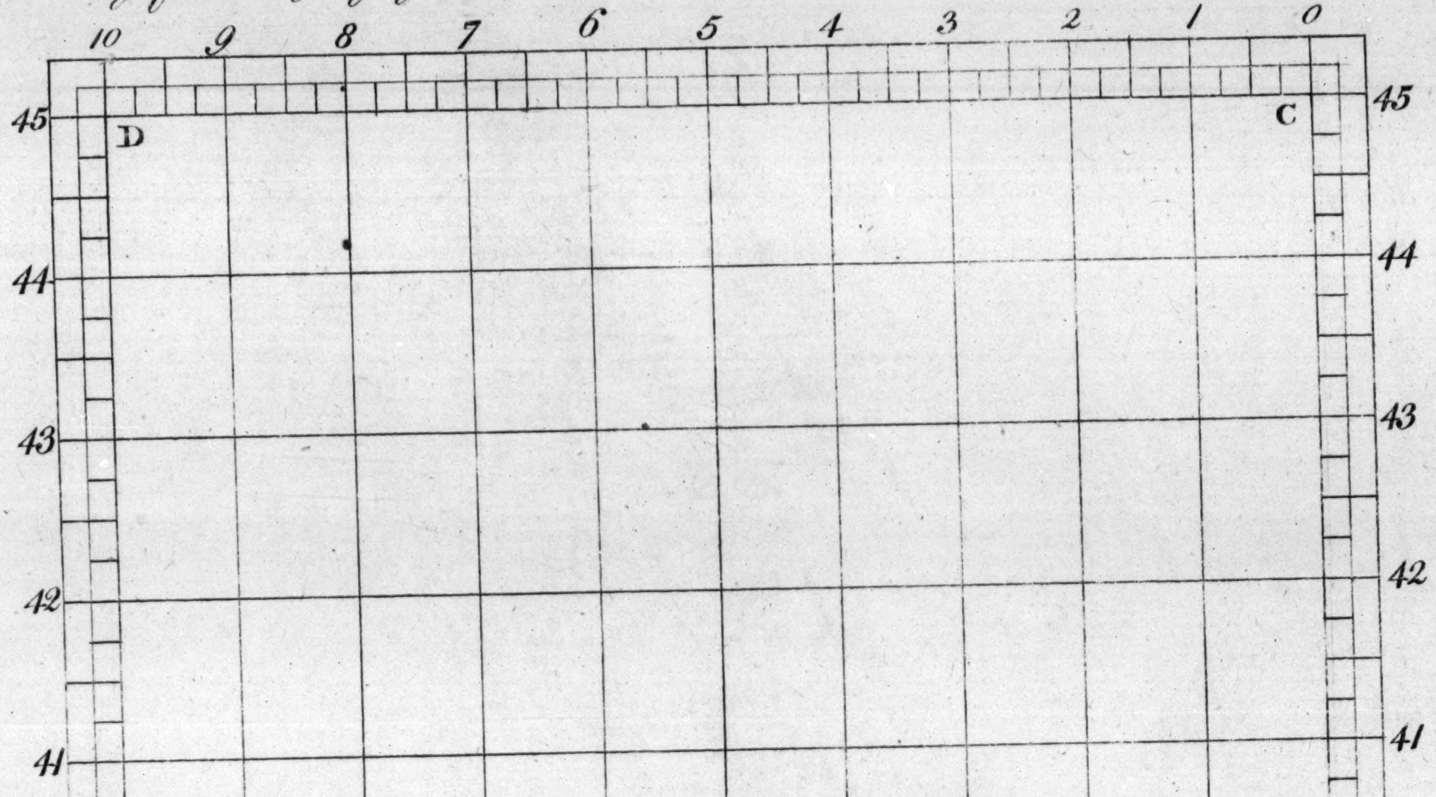
EXAMPLE.

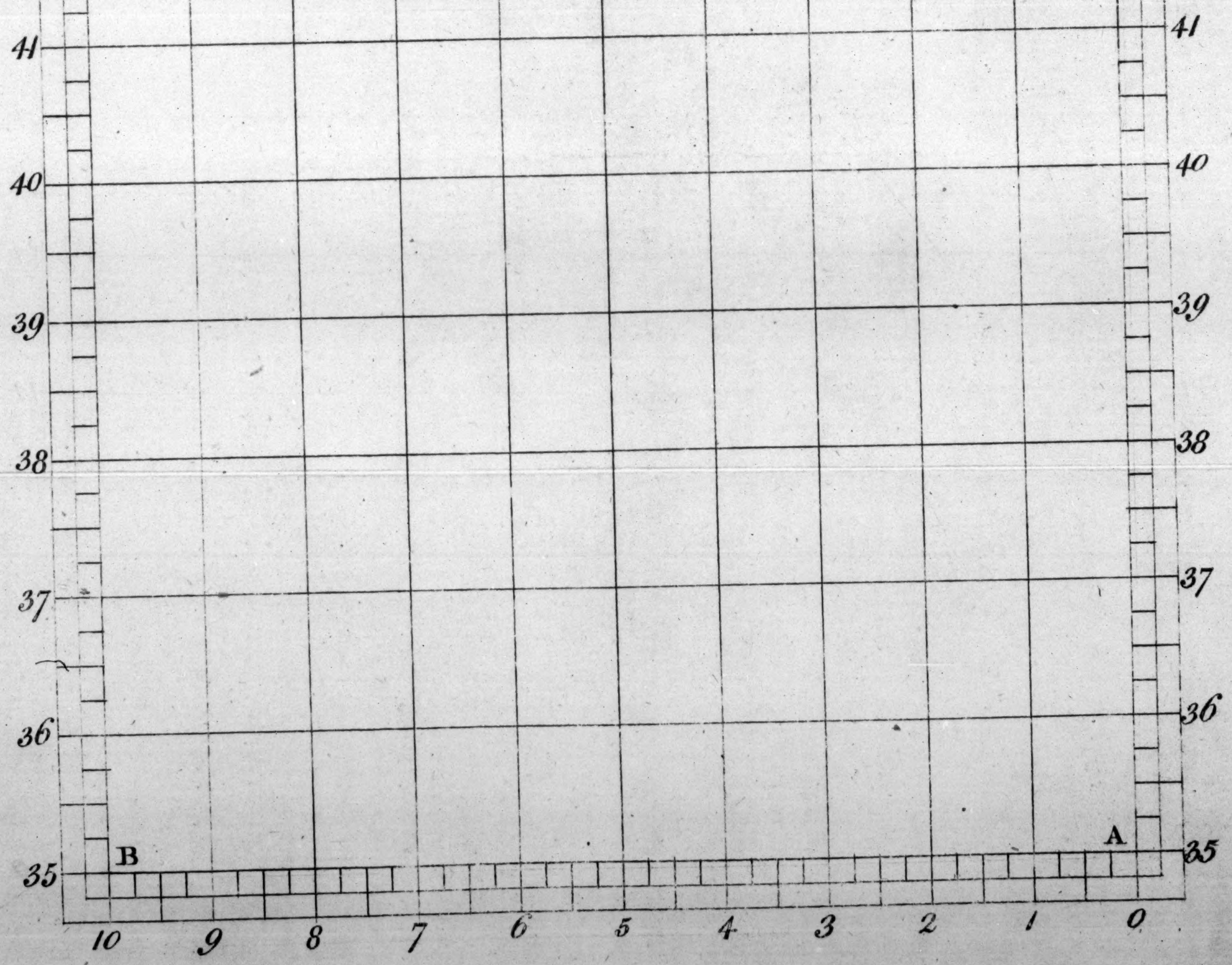
To conſtruct a Chart poſſeſſing a Space between the Parallels of 35 and 45 Degrees of North Latitude, and containing 10 Degrees of Longitude Weſt from any Meridian at Pleaſure. *See Plate* 2.

Draw the Line A B to repreſent the Parallel of 35 Degrees, and make it equal to 10 Degrees the Difference of Longitude, taken from the Equatorial Line upon the Scale; for the Length of a Degree of Longitude in any Parallel in this Chart is equal

A. Wrights Chart between 35 and 45 deg. N.º Lat. and containing ten deg. West-Long. from any Aſsigned Meridian A.C.

Plate II.

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equal to an Equatorial Degree; erect Perpendiculars from A and B, which will be Meridians; extend the Compaſſes from 35 to 45 Deg. on the Scale-Meridional Line, and lay that Extent from A to C, and from B to D; draw the Parallel D C, which concludes the Frame or Limits of the Chart; the Lines A B and C D, the Diff. of Long. muſt be divided into ten equal Parts, or taken from the Equatorial Line, as before directed; the Halves and Quarters of Degrees muſt be ſet off accordingly: Then for the Parallels, obſerve the following general Directions; Take in your Compaſſes an Extent from the leaſt Latitude to the greateſt, wanting one, that is, in this Caſe from 35 to 44 Deg. on the Mer. Line on the Scale, and lay it from A to 44, and from B to 44 on the Meridians A C and B D, and draw the Parallel 44, 44; then take an Extent from the leaſt Lat. as before, to the next inferior, in the preſent Inſtance from 35 to 43 Deg. and draw the Parallel 43, 43; then from 35 as a conſtant Number or Point, to 42, and draw 42, 42; proceed in like Manner for 41, 40, 39, 38, 37, 36, which finiſhes the whole Degrees; and for the Halves and Quarters proceed in like Manner, thus, viz. the Extent from the leaſt Lat. 35 as your Standard Point, to 44 $\frac{1}{2}$, laid upon your Chart Meridians A C and B D, will give you therein two Points through which 44 $\frac{1}{2}$ Deg. is to be drawn; thus go on for 43 $\frac{1}{2}$, &c. until all the Halves are laid down; in the ſame Manner from the leaſt Lat. 35 Deg. all the Quarters may be taken from the ſaid Meridional Line, and laid upon the two gratuated Meridians of the Chart, which being carefully done, you will have a true Nautical Plan for the Voyage; and if it be required to have the Sea Coaſts and Iſlands which are ſituate within the Compaſs, or Limits of that Plan, this may be done from their Latitudes and Longitudes, as aforeſaid. — And to determine the Courſes or Bearings of Places from one another, you need only deſcribe a Circle, or two, where moſt convenient or vacant; quarter and ſubdivide each into 32 equal Parts, draw Lines from the Center through each of the Parts in the Periphery, (but great Accuracy and Care muſt be taken) and then you will have a proper Compaſs or Fly, whereby you may determine the Bearings, and thus render your Nautical Planiſphere compleat.

Variety 3. To conſtruct a true Nautic Chart from the aforeſaid Lines of Mer. Parts, but upon a larger Scale, ſuppoſe from *that* of one Inch to a Degree; the Chart extending from

30 to 37 Degrees of North Latitude, and containing 6 Degrees of Longitude Weſt from any aſſigned Meridian. See *Plate* 3.

Draw the Line A B to repreſent the Parallel of 30 Deg. and make it equal to 6 Degrees, the Diff. of Longitude, that is, take ſix equal Parts or Inches from *that* Scale on the Back-face of the Gunter, and lay it from A to B, at which Points erect Perpendiculars, and draw the two extream Meridians A C and B D, as before; upon which Meridians lay off the Meridional Diff. Lat. in the Manner following, that is, take the Extent of the Compaſſes from 30 to 37 Degrees on the Scale Mer. Line, lay that upon the Equatorial, or Line of equal Parts, thereunto annext and belonging; mark preciſely the Number and Decimals anſwering thereto, and then take the ſame Number from the Inch Line, or Scale, firſt intended to limit the Size of the Chart, and lay this Diſtance upon the Meridians A C and B D, will give the Point C and D truly, through which the Parallel of 37 Deg. is to be drawn; and by this Means you may adjuſt the Mer. Parts on this Scale to any other you pleaſe: For it may be reduced to this Proportion, viz. as the Number of equal Parts on the Equatorial Line, is to the Extent or Length of the Meridional Parts, or Mer. Diff. Lat. thereunto belonging; ſo is the ſame Number of equal Parts taken from any Scale greater or leſs, to a fourth Proportional, or true Length of the Mer. Diff. Lat. ſuitable to *that* Scale: In the preſent Caſe, take the Extent of the Compaſſes from 30 to 37 Deg. on the Mer. Line, lay that Extent upon the Equatorial Line annext, you will find it reaches to 8.4, then the ſaid Number 8.4 taken from the Inch Scale of equal Parts on the oppoſite Side of the Gunter, will be the proper Extent or Length of the Mer. Diff. Lat. A C and B D or 7 Deg. between the Lat. of 30 and 37 Deg. applied to a Scale of one Inch to a Degree. Having thus framed or limited your Planiſphere, you proceed to lay off the other Parallels of Latitude, viz. 36, 35, 34, &c. thus, take in your Compaſſes an Extent from 30 to 36 Deg. on the Mer. Line, lay that on the annext Equatorial, it produces 7.2 nearly, then 7.2 bare taken from the aforeſaid Inch Scale, will give the Points of Section in the Meridian on the Chart, for the Parallel of 36 Degrees. Again, for 35 Deg. The Extent from 30 to 35 Deg. will reach on the Equatorial Line to 5.9, this taken from the ſaid Inch Scale, gives the Length or Point of Section for the Parallel of 35 Deg. and in like Manner proceed for all the reſt, and the Halves and Quarters may alſo be eaſily laid off by obſerving the ſame Directions; the mapping Part and Rhomb Lines may be ſupplied, if needful, by attending to the Direction given in the laſt Variety, and I hope that the whole of the Inſtructions here given, will be found ſufficient to enable the young Mariner to conſtruct his own Charts for any Voyage, and in any Manner or Size he pleaſes.

PLANE TRIGONOMETRY OBLIQUE.

PROBLEM XXII.

Two Sides and their contained Angle given, to find the other Angles and third Side.

Axiom. In all Plane Triangles, as the Sum of the two Sides, is to their Difference; ſo is the Tangent of the half Sum of their two oppoſite unknown Angles, to the Tangent of their half Difference. Then,

Add the ſaid half Difference to their half Sum, gives the greater Angle, and by Subtraction you have the leſs.

EXAMPLE.

In an oblique Triangle B C D.

Side { B C = 109, B D = 76 } Miles } given; the Angles and third Side required.
Angle CBD = 101° 30′

Preparative Work.

Side { B C = 109 — Sum of the three Angles = 180°
B D = 76 — Subt. the given Angle = 101 30

Sum of Sides = 185 — The two oppoſite Angles { Sum = 78 30
½ Sum = 39 15

Their Differ. = 33

Then,

As Sum BC and BD ·· Diff. BC and BD : : T. ½ Sum Ang. ·· T. ½ Diff.
185 Miles ·· 33 Miles : : T. 39° 15′ ·· T. 8° 17′
½ Diff. Angles = 8 17

Add is the greater Angle = 47 32

Sub. is the leſſer Angle = 30 58

PROBLEM XXIII.

Three Sides given to find an Angle. *Pl.* 1. *Fig.* 2.

Axiom. From the half Sum of the three Sides, ſubtract each Side (but firſt that Side which is oppoſite to the Angle required, then the reſt) ſeverally, noting their Remainders. Then, the Proportion will be, As the Product of the half Sum and firſt Remainder, is to the Product of the other two Remainders; ſo is the Square of Radius, to the Square of the Tangent of half the Angle oppoſite to the firſt Remainder.

EXAMPLE.

In an oblique Triangle B C D.

The Side { B C = 105, B D = 85, C D = 50 } Leag. given. The Angle C B D required.

Preparative Work.

Side { B C = 105
B D = 85
C D = 50 }

Sum of Sides = 240

½ their Sum -- 120

From ½ Sum Sub. each Side, there remains { 70, 35, 15 }

Half Sum = 120
Multip. by 1ſt. Rem. 70
Product = 8400

Second Rem. = 35
Third Rem. = 15
Product = 525

By LOGARITHMS.

Then, As Prod. $\frac{1}{2}$ Sum and 1ſt. Rem. ·· Prod. other 2 Rem. : : Sq. Rad. ·· Sq. Tan. $\frac{1}{2}$ the Angle opp. 1ſt. Rem.

8400 Leag. ·· 525 Leag. : : 20.000.000 ·· 18.795880

$14^{\circ}\ 2'$) $\frac{1}{2}$ Sum = 9.897940
14 2

Double is = 28 4 = Angle required.

By the Slider.

Set 8400 Num. on the fixt Piece, againſt 525 Num. on the Slider, then againſt Tang. 45 on the fixt Piece will be found 3° 32′ Tan. on the Slide, take half the Diſtance between that Point and Tang. Radius on the Slide, the middle Point will be found at Tang. 14 Deg. which doubled, is 28 Deg. the ſame as before by Logarithms, wanting four Minutes.

With Compaſſes wholly.

The Extent from 8400 to 525 on Numbers, will reach from Tan. Radius to T. 3° 32′, half the Extent from thence to Radius (which is the ſame as extracting the Square Root) points out 14 Degrees as before, for half the Angle, this doubled, gives 28 Deg. for the whole Angle required.

It happens ſometimes that the Sides run large, and then the Products of the half Sum and Remainders are high and troubleſome; to avoid which, as well as the Multiplications herein concerned, a Method has been deviſed to perform this Problem by the ſimple Addition of four Terms, viz. the Co. Ars. of the half Sum and firſt Remainder, and the Logarithms of the other two Remainders, which will give the Square of the Tang. this divided by two, gives the Tangent; for to divide a Log. by 2, gives the Square Root, and dividing by 3, gives the Cube Root of any Numbers correſponding with thoſe Logarithms: I ſhall here repeat the Work of this Queſtion by the Method now mentioned, and point out the Reaſon of the Operation, naturally ariſing from the Axiom itſelf.

Note, The Co. Ar. or Complement Arithmetical of any Number is the Remainder when ſubtracted from Logarithmic Radius, and when Radius is not the firſt Term in any Proportion, then take the Co. Ar. of the firſt Term, and add it to the Logarithms of the ſecond and third Terms (inſtead of ſubtracting in the uſual way) and from the Index or Characteriſtic of their Sum, cancel or ſubtract 10 or 20, the remaining Figures is the Log. of the fourth Term ſought. By diſpoſing of the Members contained in the Terms of the foregoing Proportion ſingly, the whole Matter will be ſufficiently illuſtrated.

As Prod. ½ Sum and 1ſt. Rem. ·· Prod. other 2 Rem. : : Sq. Radius ·· Sq. Tang. ½ Angle opp. 1ſt. Rem.
120 × 70 ·· 35 × 15 : : 20.000.000

	Log.	1.544068	*	38.795880	
	Log.	1.176091		20.	Sub. for D. Co. Ar.
120	Co. Ar.	7.920819		———	
70	Co. Ar.	8.154902	½)	18.795880	
		———		———	
		38.795880	*	9.397940	(14° 2′ 14 2
					——— 28 4 as before.

Now, as Addition of Logarithms performs the Multiplication of their correſpondent natural Numbers, and as the ſecond Term conſiſts of two Members to be multiplied together, therefore their two reſpective Logarithms being taken ſingly, anſwers the ſame Purpoſe fully, and ſaves the Trouble of Multiplication, or finding a Product. And whereas the firſt Term alſo conſiſts of two Members to be multiplied for a Product, theſe may be treated in like Manner; their Comp. Arith. or Co. Ars (as being the firſt Term) may be alſo taken here to Advantage, and added to the others, the Sum of which will be the Tang. Log. of the fourth Term ſought, but there being two Numbers or Members in the firſt Term, and theſe Co. Ar'd. ſeparately, a double Radius, or 20, that is, the Square of Radius Logarithmically, muſt be cancelled or ſubtracted from the Index, and the Remainder is the proper Log. of the fourth Term; but whereas the third Term conſiſts of the Square of Radius to be added, and the firſt Term on account of the double Co. Ar. requires the Square of Radius to be ſubtracted, therefore reject the third Term out of the Operation, and then its Equivalent (on account of the firſt Term to be ſubtracted) will be annihilated in Courſe; hence it evidently appears, that the Sum of the two Co. Ars. and the other two Logarithms, give the Log. of the fourth Term required.

This Method is uſeful and conciſe, as it finds an Angle at one Operation, but as it is not ſo applicable to the Scale, I ſhall here give another Axiom, whereby (the ſame Things given) an Angle may be readily found with that Inſtrument, by two Proportions.

PROBLEM XXIV.

Three Sides given to find an Angle.

Axiom or *Rule*. As the longeſt Side, is to the Sum of the two ſhorteſt; ſo is the Difference of the two ſhorteſt, to the Difference of the Segments of the Baſe, or longeſt Side.

Preparative to this Proportion, let fall a Perpendicular, in this Caſe D A (from the Angle oppoſite) to the longeſt Side, which divideth it into two Segments, and the oblique Triangle into two right-angled Triangles.

EXAMPLE.

The ſame as in the laſt Problem, viz.

$$\left.\begin{array}{l} BC = 105 \\ BD = 85 \\ CD = 50 \end{array}\right\} \text{Leag. given.} \quad \text{The Angle } CBD \text{ required.}$$

$$\text{Shorteſt Sides} \left\{\begin{array}{l} BD = 85 \\ CD = \underline{50} \end{array}\right. \quad \textit{Plate 1. Fig. 2.}$$

Sum of two ſhorteſt = 135

Diff. of two ſhorteſt = 35

Then, As B C the longeſt ·· Sum of B D and CD : : Diff. B D and CD ·· B E the Diff. Segments.

105 ·· 135 : : 35 ·· 45 Leag.

Longeſt Side B C = 105
Diff. of Segm. B E = 45

$$\left.\begin{array}{r} \text{Added is} = 150 \\ \text{Subtract. is} = 60 \end{array}\right\} \text{the half is} \left\{\begin{array}{l} 75 = BA \text{ greater} \\ 30 = AC \text{ leſs} \text{ —} \end{array}\right\} \text{Segment.}$$

Now the two Segments, or Sides of the two right-angled Triangles being found, the Angles are readily determined by making either of the Sides Radius; for Inſtance in this Caſe, to find the Angle C B D.

1. The Hypothenuſe B D Radius.

As B D ·· Radius : : A B ·· S. A D B.
85 ·· S. 90 : : 75 ·· S. 61° 56′
Subt. from 90 00

Remains Angle C B D = 28 4 as before.

2. Leg. A B Radius.

As A B ·· Radius : : B D ·· Sec. C B D or A B D.
75 ·· Sec. 0 : : 85 ·· Sec. 28° 4′ as above.

This laſt by the Slider.

75 being ſet againſt Radius, 85 will ſtand againſt blank beyond Radius; therefore ſet 85 on the Slide againſt Sine 90, that is, Sec. 00, and againſt 75 on the Num. you will find on the Sines, reckoning backwards from Radius, Sec. 28° 4′.

Or with Compaſſes.

The Extent from 75 to 85 on Num. will reach on the Sines backwards as before, to Sec. 28° 4′, or 61° 56′ on the Sines forwards; hence it appears that the two laſt Proportions are the ſame in Fact, though not in Form; Secants being changed into Sine Compliments.

I have rather enlarged upon this Problem, as there is ſome Conſimilarity (reſpecting the Mode of Operation) with *that* of a Spheric Triangle, where the three Sides are given; a Problem ſo frequent, and practically uſeful in Aſtronomy.

The Uſe of the Scale in Aſtronomy.

PROBLEM XXV.

The Sun's Place in the Ecliptic, and his greateſt Declination given; to find his right Aſcenſion, and preſent Declination.

EXAM-

A. Wrights Chart between 30 and 37 deg. N° Lat. and containing six deg. W. Long. from any aſsigned Meridian A.C. taken from an enlarged Scale.

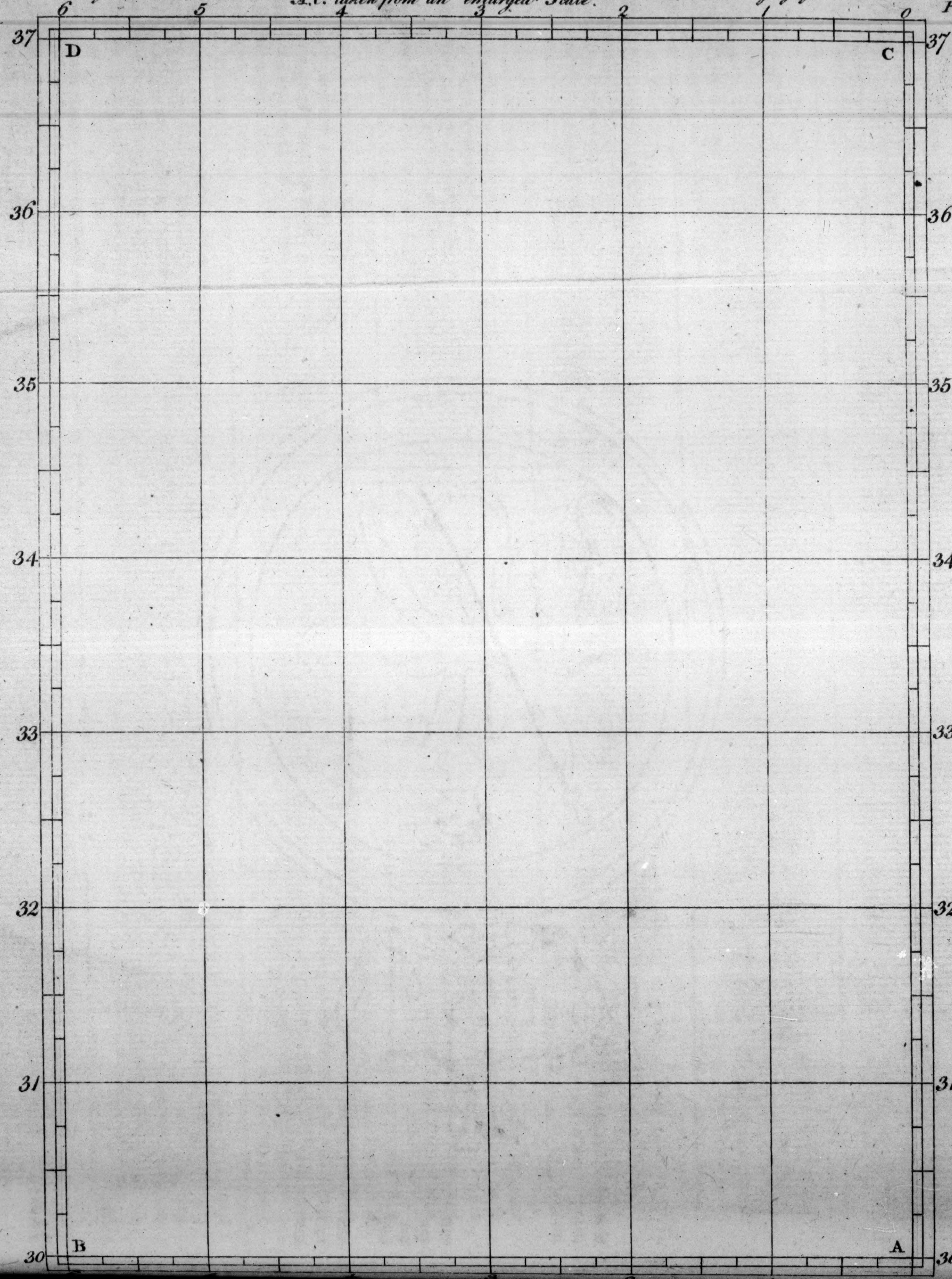
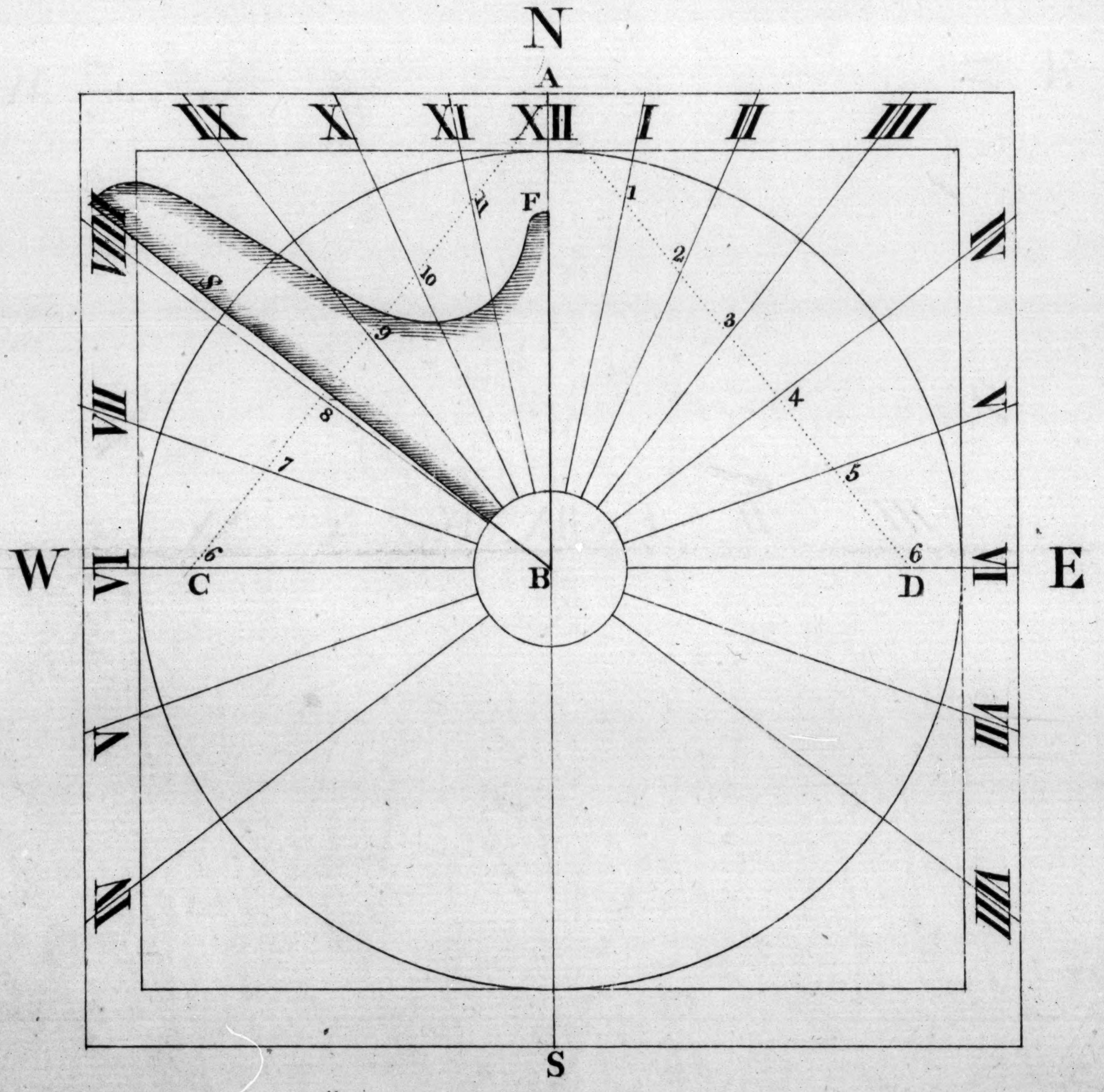


Plate IV.


N
A
IX
X
XI
XII
I
II
III
F
11
10
9
8
7
6
C
1
2
3
4
5
6
D
IIII
V
VI
VII
VIII
VIII
VII
VI
V
IIII
W
B
E
S

EXAMPLE.

The Sun being in 24° 15′ of Taurus; What is his right Aſcenſion and Declination?

The Proportion to find the Declination, is,

Radius ·· S. ⊙'s Long. : : S. ⊙'s gr. Decl. ·· S. preſent Decl.
S. 90d. ·· S. 54° 15′ : : S. 23° 29′ ·· S. 18° 52′ N.

With the Slider.

Set S. 90 on the fixt againſt S. 54° 15′ on the Slide; then againſt S. 23° 29′ on the fixt (directed by the Braſs Index) you will have S. 18° 52′ on the Slide.

With Compaſſes.

The Extent from Radius to S. 54° 15′ will reach the ſame way, from 23° 29′ to S. 18° 52′.

For the right Aſcenſion.

T. Co ⊙'s Long. ·· Radius : : S. Co. ⊙'s gr. Decl. ·· T. right Aſ.
T. 35° 45′ ·· T. 45° : : S. 66° 31′ ·· T. 51° 52′

The third Term being greater than the firſt, the fourth will be more than Radius, or will extend beyond T. 45°, therefore, ſet S. 66° 31′ on the Slide, againſt Radius on the fixt Piece; then againſt T. 35° 45′ on the Slide, you will find 51° 52′ on the fixt Tang. Line, reckoning backwards from 45.

With the Compaſſes.

The Extent from S. 66° 31′ to T. 35° 45′, will reach from S. 90° to T. 51° 52′, as before.

PROBLEM XXVI.

The Latitude of a Place, and the Sun's Declination given, to find the Sun's Amplitude and Aſcenſional Difference?

EXAMPLE.

Latitude 51° 31′ } N. given. Amplitude } required.
Sun's Decl. 23 29 } Aſcen. Diff.

Proportion for the Amplitude, is,

S. Co Lat. ·· Radius : : S ☉'s Decl. ·· S. ☉'s Ampl.
S. 38° 29′ ·· S. 90° : : S. 23° 29′ ·· S. 39° 49′ N.

Note, The Amplitude is always of the ſame Name with the Declin.

For the Aſcenſional Diff.

Radius ·· T. Latitude : : T. ☉'s Decl. ·· S. ☉'s Aſc. Diff. or hour
T. 45° ·· T. 51° 31′ : : T. 23° 29′ ·· S. 33° 8′. [from Six.

To work theſe Proportions by the Scales, needs no Explanation.

The Aſcenſional Difference being reduced into Hours and Minutes, and added to, and ſubtracted from Six Hours, the one is the Time of Sun Riſing, the other of Setting: *Note*, that 15 Deg. of the Equinoctial is equal to one Hour of Time; one Deg. is four Minutes, and 15 Min. of a Deg. one Min. of Time. — Hence the Aſcenſional Difference here found

=33° 08′ is in Time=2h 12′ 32″
which added to 6 0 0

is Sun Setting=8 12 32 } leng. of { Day is=16h. 25′ 4″
Subtr. is the Riſing=3 47 28 } { Night is= 7 34 56

Note, The Lat. and Declin. both N. or both S. the Sun riſes before, and ſets after Six,—bnt if one be N. and the other S. then he riſes after, and ſets before Six.

PROBLEM XXVII.

Three Sides of a Spheric Triangle given, to find any of the Angles.

Axiom. As the Rectangle or Product of the Sines of the two Sides, containing the Angle required, is to the Square of Radius; ſo is the Rectangle or Product of the Sine of half the Sum of the three Sides, and of the Sine of the Difference between the ſaid half Sum and the Side oppoſite to the Angle required, to the Square of the Sine Complement of half the contained Angle ſought.

Note, That this Proportion may be performed by Logarithms, very conciſely, by taking Arithmetical Compliments of the two Numbers which conſtitute the firſt Term, and by totally cancelling or expunging Radius the ſecond Term, for then you have only the Addition of the two Co. Ars to the two Sine Logarithms belonging to the third Term, this Sum is the Square of the Sine Compl. as above.

The Reaſon of this Contraction will better appear by duly conſidering what is ſaid in the Explanation of Problem XXIII.

EXAMPLE.

The Latitude of a Place, the Sun's Declination and Altitude given, to find his Azimuth.

The { Latitude 51° 30′ / ⊙'s Decl. 15 10 / ⊙'s Altit. 11 30 } North. His Azimuth in the Morn. or Aftern. is required?

Preparative Work.

Comp.Lat.	38° 30′	Containing Sides	S.Co.Ar.	0.205850
Comp.Alt.	78 30		S.Co.Ar.	0.008807
⊙'s Pol.Diſt.	74 50			
Sum is	191 50	½ Sum Sides ſuppl, 84° 05′	S. Log.	9.997680
½ Sum is	95 55	Remainder is 21 05	S. Log.	9.555971
Rem. is	21 05		Sum of the 4 Logar.	19.768308

49° 59′ = S. ½ Sum = 9.884154

Comp. of ½ the Angle required is = 49° 59′ or 90 00 − 49 59 = 40 01 Rem. ½ An.

49° 59′ + 49 59 = 99 58

40 01 × 2 = 80 02 the Azim.

Subtracted from 180 00

Sun's Azim. from N,E.ly, or N,W.ly = 80 02 according as it is Morn. [or Aftern.

But this Axiom may be reſolved into two Proportions, and readily performed on the Scale; the firſt by finding a fourth

Sine or Proportional to Radius and the two containing Sides, thus,

As Radius it to the Sine of one of the containing Sides, ſo is the Sine of the other containing Side, to a fourth Sine; and then ſecondly, As this fourth Sine or Proportional found, is to the Sine of half the Sum of the three Sides, ſo is the Sine of the Remainder, to a ſeventh Sine; right againſt which on the Line of Verſed Sines on the Scale, you will have the Degrees and Minutes of the Angle required.

In the preſent Caſe to find the Azimuth, thus,

As Radius ·· S. co. Latitude : : S. co. ☉'s Altit. ·· fourth Sine.
S. 90 ·· S. 38° 30' : : S. 78° 30' ·· S. 37° 35'

Then,

As fourth Sine ·· S. ½ Sum Sides : : Remainder ·· ſeventh Sine.
S. 37° 35' ·· S. 95° 55' : : S. 21° 05' ·· S. 35° 55' againſt
180 00 [which on the Line of
[Verſed Sines is S. 80° 02'
its Supp. 84 05 [the Sun's Azim. as before.

By letting fall a Perpendicular from an oppoſite Angle to one of the containing Sides, (when three Sides are given to find an Angle) and thereby obtaining two right-angled Triangles; ſundry Axioms or Rules have been inveſtigated, and may be uſed at Pleaſure, but the foregoing is rather adopted here, as having been long in Practice, and is as eaſy and conciſe as any, both Logarithmically and by this Scale.

PROBLEM XXVIII.

The Latitude of a Place, the Sun's Declination and Altitude given; to find the Hour of the Day.

As the laſt Example is eſſential in Practice at Sea, in order to determine the true Azimuth, upon which the Knowledge of the Variation of the Compaſs depends; ſo is this no leſs neceſſary for the Determination of the preciſe Time when Obſervations are made, the Accuracy of your Longitude account hereupon ſo materially depending.

EXAM-

EXAMPLE.

The ſame as before, viz.

The { Latitude 51° 30′ } North. The Hour of the Day
☉'s Declination 15 10 } is required?
☉'s Altitude 11 30 }

Preparative Work.

Comp. Lat. 38° 30′ } Containing Sides { S. Co. Ar. 0.205850
☉'s Polar Diſt. 74 50 } { S. Co. Ar. 0.015397
Comp. Altit. 78 30

Sum is = 191 50

½ Sum is = 95 55 its Supplem. = 84° 05′ S. Log. 9.997680

Remainder = 17 25 S. Log. 9.476133

Sum of the 4 Logs. = 19.695060

S. ½ Sum = 9.847530

S. Comp. of ½ Angle = 44° 45′ } doubled
44 45 }

89 30 which
ſubtract from 180 00

Rem. 90 30 = 6. h 02 m. the Hour from Noon.

Proportions for the Scales.

1. As, Radius ·· S. Co. Latit. : : S. ☉'s Pol. Diſt. ·· 4th Sine.
S. 90° ·· S. 38° 30′ : : S. 74° 50′ ·· S. 36° 56′

2. As, 4th Sine ·· S. ½ Sum Sides : : S. Remainder ·· 7th Sine.
S. 36° 56′ ·· S. 95° 55 : : S. 17° 25′ ·· S. 29° 42′.

Againſt which on the Line of Verſed Sines is 90° 30′, or 6h. 2m. from Noon, P.M. or 58 min. after 5 in the Morning, the preciſe Time required.

PROBLEM XXIX.

Two Sides and the included Angle of a Spheric Triangle, given, to find the other two Angles, and the third Side.

Axiom, Rule, or Proportion.

Firſt, As the Sine of half the Sum of the two Sides (containing an Angle) is to the Sine of half their Difference; ſo is the Tangent Comp. of half the contained Angle, to the Tang. of half the Difference of the other two Angles.

Secondly, As the Sine Comp. of half the Sum of the two Sides (containing an Angle) is to the Sine Comp. of half their Difference; ſo is the Tang. Comp. of half the contained Angle, to the Tang. of half the Sum of the other two Angles; then the half Diff. added to the half Sum, gives the greater, and ſubtracted gives the leſſer.

EXAMPLE.

Pl. 1. *Fig.* 3. 4.

In the Oblique Spheric Triangle A C D.

Given { Side A C = 34° 06′ / Side A D = 65 20 / Angle CAD = 30 46 } The Angles and other Side required.

Preparative Work.

Side { A C = 34° 06′ / A D = 65 20 } Angle C A D = 30° 46′
the half is = 15 23
Sum of Sides = 99 26 / Diff. of Sides = 33 14 } the ½ { Sum = 49 43 / Diff. = 15 37 } Comp. { 74° 37′ / 40 17 / 74 23

The Proportion may now be formed, and eaſily worked by the Scale.

Then, As,

S.½Sum Sides ·· S.½Diff.Sides : : T.Co.½ con.Ang. ·· T.½Diff.otherAn.
S. 49° 43′ · S. 15° 37′ ·· Tan. 74° 37′ · Tan. 52° 03′

Again, As,

S.Co.$\frac{1}{2}$SumSi. · S.co.$\frac{1}{2}$Diff.Si. . . T.co.$\frac{1}{2}$ con.Ang. · T.$\frac{1}{2}$ Sum of Ang.
S. 40° 17′ · S. 74° 23′ . . Tan. 74° 37′ · Tan. 79° 32′
. Add the $\frac{1}{2}$ Diff. above 52 03

Sum is the gr. Angle 131 35

Subt. is the leſs 27 29

Note, If the Sum of the two containing Sides exceeds a Semi-circle, then ſubtract each Side from 180 Degrees, and proceed with theſe Remainders, as with the Sides given; the Proportions then produce the Supplement of the Angle required, to a Semi-circle.

Having found theſe Angles, the third Side is readily obtained in the uſual Manner, by oppoſite Sides and Angles; but if this third Side ſhould be wanted without the Angles, it may be done more conciſely by letting fall a Perpendicular from one End of the leſs given Side to the greater, and thereby reducing the Oblique Triangle into two Right-angled Triangles; which done, the third Side may be found at two Proportions, by Lord *Napier's Catholic* Propoſition, and is thus,

Firſt,

As Radius ·· S Co.conta.Ang. : : T.leſs given Side ·· T. 4th Ar.
S. 90 ·· S. 59° 14′ : : T. 34° 06′ ·· T. 30° 11′
which being ſubtracted from 65 20 = Side [AD.

Rem. the *Reſidual Arc* 35 09

Note, If the contained Angle be acute, ſubtract the fourth Arc from the greater given Side, but when obtuſe from the Supplement thereof to a Semi-circle, the Remainder is called the *Reſidual Arc*.

Then, ſecond,

As, S.Co. 4th Ar. ·· S.Co.Reſid.Ar. : : S. Co. leſs given Side ·· S. Co. of Side required.
S. 59° 49′ ·· S. 54° 51′ : : S. 55° 54′ ·· S. 51° 34′
Subtract from 90 00

Rem. 38 26 = CD.

Note, When the contained Angle and Reſidual Arc are both more or leſs than a Quadrant, or 90 Degrees, the Side ſought is leſs than a Quadrant, but when one is more and the other leſs, it is more than a Quadrant.

It may not be unacceptable to ſome Readers to have the two laſt Proportions demonſtrated, which ſhall be done in the following Manner. (See *Fig*. 3. and 4. *Plate* 1.)

Let fall the Perpendicular C Y, which will divide the Oblique Triangle given, into two Right-angled Triangles, A Y C and D Y C, and firſt, to find A Y the fourth Arc.

Firſt, T. Co. A C .. Radius : : S. Co. C A Y .. T. A Y.

But Radius being a mean Proportional between the Co. Tangent and Tangent, therefore change the Terms, and the Proportion will ſtand right thus,

Radius .. T. A C : : S. Co. C A Y .. T. A Y.
Radius .. T. leſs given Side : : S.Co.contained An. .. T. 4th Arc.

Second, Suppoſe the common Perpendicular and Baſes of each of theſe two Right-angled Triangles to be given reſpectively, to find the other two reſpective Sides, Then,

In Triang. AYC, As, Rad. .. S.co.CY : : S.co.AY .. S.co.AC.
In Triang. DYC, As, Rad. .. S.co.CY : : S.co.DY .. S.co.CD.

But Numbers or Terms which are Proportionals to one and the ſame Terms, are Proportionals to eath other; And, as the two firſt Terms here, in both Proportions are the ſame, *they may be cancelled*, and the laſt Terms muſt reſpectively be Proportionals to each other alſo.— That is,

As, S. Co. A Y .. S. Co. D Y : : S. Co. A C .. S. Co. C D.
Or, As, S. Co. 4th Arc .. S. Co. Reſidual Arc : : S. Co. leſs given Side .. S. Co. Side required. *Q. E. D.*

PROBLEM XXX.

Two Angles and an interjacent Side given; the other two Sides, and the third Angle required.

Axiom or Proportion.

Firſt, As the Sine of half the Sum of the two Angles given, is to the Sine of half their Difference; ſo is the Tangent of half the interjacent Side, to the Tangent of half the Difference of the other two Sides.

Secondly, As the Sine Complement of half the Sum of the two given Angles, is to the Sine Complement of half their Difference; ſo is the Tangent of half the interjacent Side, to the Tangent of half the Sum of the Sides; then the half Diff. added to the half Sum, gives the greater Side, and ſubtracted, gives the leſs, as before.

EXAMPLE. *Pl.* I. *Fig.* 3, 4.

In the Oblique Spheric Triangle.

Given	Angle	A C D —	131°	34′	The Sides and third Angle required, or C A D.
	Angle	A D C —	27	30	
	Side	C D —	38	28	

Note, If the Sum of the two given Angles exceed 180 Deg. then ſubtract each given Angle from 180 Deg. and proceed with the Remainders as with the Angles given; the Operation will then produce each required Side's Supplement to the Semicircle.

The preparative Work muſt here be done, ſomewhat in like Manner as in the laſt Problem, and then the Proportion may be eaſily ſtated and worked Inſtrumentally, or otherwiſe, and the third Angle found by oppoſite Sides and Angles.

But if this third Angle ſhould be required without the Sides, it may be found more readily by letting fall a Perpendicular from the greater given Angle to its oppoſite Side, and thereby reducing the Oblique Triangle into two Right-angled Triangles, when the third Angle may be found, by Lord *Napier's Catholic* Propoſition, at two Proportions, in like Manner as before, and is thus,

1. As, Radius ∙∙ S. Co. of the interjacent Side : : Tan. leſs given Angle ∙∙ Tan. of 4th Arc, then, if the interjacent Side be more than a Quadrant, ſubtract the fourth Arc from the greater given Angle; but when it is leſs, then from the Sup-

plement

plement thereof to 180 Deg. the Remainder is the *Reſidual* Arc. Then,

As, the Sine Com. of 4th Arc ·· Sine Com. of Reſidual Arc : : S. Co. leſs given Angle ·· S. Co. of Angle required.

Note, When the adjacent Side and Reſidual Arc are both more or leſs than 90 Deg. the Angle ſought is acute, but when one is more, and the other leſs, it is obtuſe; in this Caſe it is acute, and is equal to 30° 47′ = the Angle C A D.

Theſe laſt two Proportions will admit of the ſame Kind of Demonſtration, as is uſed in the laſt Problem; the Performance of which, as well as the Operation of this Example given, is left for Practice.

Thoſe Rules duly obſerved, and the true State of the Proportions obtained, all Difficulties touching Inſtrumental Operations, in every Branch of Science, will be totally removed and done away.

PROBLEM XXXI.

The Moon's apparent Altitude, and alſo her Horizontal Parallax given, to find her Parallax in Altitude.

EXAMPLE.

June the 5th, 1771, The apparent Altitude of the Moon's Center was obſerved to be 44° 07′, when her Horizontal Parallax was 54° 54: Required the Parallax in Altitude?

Note, Parallax is that Arch of a great Circle which paſſeth through the Zenith and true Place of the Sun, Moon, or Star, and intercepted between the true and apparent Place; becauſe the true Place is ſuppoſed to be beheld from the Earth's Center; but the apparent Place from the Superficies, and that Difference is the Arch above noted, or the Meaſure of the Angle of the Parallax.

The Horizontal Parallax is the greateſt, and as the Celeſtial Object increaſes in Altitude, it diminiſhes in Parallax, and when in the Zenith, the Parallax totally vaniſhes; therefore as the Zenith Diſtance decreaſes, ſo does the Parallax, and that in Proportion to the whole Sine or Radius.

Hence the Proportion will be,

As, Radius .. Sine Moon's Zenith Diſt. : : Horizontal Parallax .. Parallax in Altitude.

S. 90° .. S. 45° 53′ : : 54′ 54″ on Numbers .. 39′ 25″

Therefore by the Scale, Set Radius S. 90° on the fixt Part, right againſt the Sine of the Zenith Diſt. on the Slide; then againſt the Minutes or Seconds of the Horizontal Parallax taken on the fixt Part, on the Line of Numbers, you will have the Minutes or Seconds of the Parallax in Altitude, on the Line of Numbers on the moveable Part or Slider.

Or with the Compaſſes.

The Extent from Radius to Sine of the Moon's Zenith Diſtance being laid the ſame Way from the Horizontal Parallax, will reach to the Parallax in Altitude required.

PROBLEM XXXII.

Given the Altitudes of the Sun and Moon, and their apparent Diſtance; Required their true Diſtance.

General Directions.

With the apparent Diſtance, and the two Zenith Diſtances, conſidered as the three Sides of a Spheric Triangle, find the Angle at the Vertex, by Problem XXVII. Then find the Parallax in Altitude by the laſt Problem; correct the Zenith Diſtances thereby, and alſo for Dip. and Refraction, &c. Then, with theſe Zenith Diſtances thus corrected, as the two Sides of a Spheric Triangle, and the Vertical Angle juſt found, included between them, find the third Side by Problem XXIX. which Side is the true Diſtance required.

Then, having this Diſtance, the Time at *Greenwich* may be found when the Sun and Moon had this Diſtance, which compared with the Time of the Obſervation, and the Difference reduced into Deg. Min. and Sec. determines the Difference of Longitude.

But it cannot be expected that theſe general Directions will be ſufficient for a Student altogether unacquainted with the

Manner of working theſe Kind of Obſervations, I ſhall therefore refer him to the Nautical Ephemeris, publiſhed annually by Order of the Honourable Board of Longitude, and to the further Inſtructions given in ſome of them by the Rev. *Nevil Maſkelyne*, D. D. Aſtronomer Royal, and F. R. S.

However, to adminiſter all the Aſſiſtance poſſible in this Place, and in order to point out how far this Scale may be ſerviceable therein, I have ſubjoined an Example, with the Operation at large.

PROBLEM XXXIII.

EXAMPLE. *Pl.* I. *Fig.* 5.

June the 5th, 1771, at 18h. 55′ 10″ by a Watch, which was found to be 49″ too faſt for the Sun: I obſerved the Diſtance of the Sun and Moon's neareſt Limits to be 81° 10′, when the Apparent Altitude of the Sun's Center was 25° 33′ and that of the Moon 44° 07′; required the Longitude?

Time per Watch	18h.	55′	10″
Error	00	00	49
True App Time	18	54	21

	h.	′	″
Obſ. Diſt. ☉ & ☽'s limbs =	81	10	00
Sun's Semi-Diam. =	0	15	48
Moon's Semi-Diam. =	0	14	57
Augmentation =	0	0	11
Obſerved Diſt. of Centers =	81	40	56

☉ & ☽'s App. Diſt. =	81°	40′	56″		
Sun's Zen. Diſt =	64	27	00	Contained Sides	S.Co.Ar. 0.0446927
Moon's Zen. Diſt. =	45	53	00		S.Co.Ar. 0.1439216
Sum is	192	00	56		
½ Sum is	96	00	28	Sup. is 83° 59′ 32″ S.	9.9976080
Remainder	14	19	32	Si.	9.3934545
				Sum of the 4 Logar.	19.5796768
				½ Sum 51 56 56 Si.Co.	9.7898384
				2	
The Angle ☉ Z ☽	103	53	52	= < at Vertex.	

Or, This Angle at the Vertex may be found by two Proportions on the Scale, by a ſecond Method made Uſe of in the Operation of Problem XXVII. where the three Sides are given, to find the Vertical Angle.

Again,

As, Radius ·· S. ☽'s Zen. Diſt. : : Hor. Parallax ·· Par. in Alt.
S. 90° ·· S. 45° 53′ : : 54′ 54″ ·· 39′ 25″

	° ′ ″		° ′ ″
Moon's Par. in Alt. =	00 39 25	⊙'s App. Zen. Diſt. =	64 27 00
— Refraction =	00 00 59	+ Refraction =	00 01 59
Cor. of ☽'s Zen. Diſt. =	00 38 26	Sun's true Zen. Diſt. =	64 28 59
☽'s App. Zen. Diſt. =	45 53 00	Angle at the Vertex =	103 53 52
		Subt. from	180 00 00
☽'s true Zen. Diſt. =	45 14 34	Sup. of cont. < ☽ZY =	76 06 08

In the Triangle Z ☽ Y being formed by the Perpendicular ☽ Y.

Co.T. Z ☽ ·· Radius : : Co.S. ☽ Z Y ·· Tan. Z Y.
Co.T. leſs giv. Side ·· Radius : : Co.S. con. Ang. ·· Tan. 4th Arc.
45° 14′ 34″ ·· 90 00 : : 76 06 08 ·· 13 37 00
which added to —— 64 28 59
13 37 00

Reſidual Arc ⊙ Y = 78 05 59

In the Triangle ⊙ ☽ Y.

Sine Co. 4th Arc Z Y ·· S. Co. Reſidual Arc ⊙ Y : : S. Co. leſs given Side Z ☽ ·· S. Co. required Side, ⊙ ☽.
13° 37′ 00″ ·· 78° 05′ 59″ : : 45° 14′ 34″ ·· true 81° 24′ 30″
= ⊙ and ☽'s reduced Diſt.

°	′	″		
81	24	30	= true reduced Diſtance	
82	09	52	at 18 Hours	
80	46	42	at 21 Hours	
1	23	10	Diff. in three Hours	Prop. Log. 3353
00	45	22	Diff. betw. the reduced Diſt. and next leſs	Prop. Log. 5985
01	38	12		Prop. Log. 2632
18	00	00		
19	38	12	= time at Greenwich	
18	54	21	= true Time at Ship	
00	43	51	= 10° 57′¾ Diff. of Long. Weſt.	

The Times correſpondent with the true reduced Diſtances to every three Hours, are to be found in the Nautical Ephemeris before-mentioned, for that Year; and the intermediate Time between each three Hours, is obtained from the Difference of the Diſtances of the Luminaries made within that three Hours, by the Help of the Proportional Logarithms found in the Tab. requiſite to be uſed with the ſaid Ephemeris, as may be obſerved in the Work above.

This Figure may well enough ſerve to the 32d Problem, in a double Capacity; as firſt, where the three Sides are given, to find the Angle at the Vertex ⊙ Z ☽; and ſecondly, when the neceſſary Corrections are made, having the two Sides, and the included Angle, or Angle at the Vertex, given, to find the third Side; that is, the true reduced Diſtance of the ⊙ from the ☽.

But if the Proportional Logarithms ſhould be wanting, and and the Time at *Greenwich* required without them, it may be found by the Scale; for the Proportion will be,

As, Diff. of Diſt. of ⊙ and ☽ in Deg. &c. in three Hrs ·· three Hrs. : : Diff. of reduced Diſt. and next leſs, or 18 Hours ·· Proportional intermediate Time required.

1° 23′ 10″ ·· 3h. : : 0° 45′ 22″ ·· 1h. 38′ 12″.

But theſe may be reduced into Minutes, (multiplying by 60) and the Seconds into Tenths of a Minute, and then the Proportion

portion will ſtand thus, and may be wrought on the Line of Numbers, viz.

As, 83.2m. nearly ·· 180m. : : 45.4m. ·· 98.25m. or 1h. 38′15″.

Exceeding the exact Time found before, only by three Seconds; but it is not adviſeable to truſt to the Scale in theſe curious Caſes, if it can be well avoided, as every Second of Time here makes a Difference of Longitude of 15 Seconds, or one Quarter of a Minute of a Degree; therefore it may be performed by the Rule of Three, or more conciſely by Common Logarithms.

Hence it appears, that this Scale may be uſefully applied in every Branch of Science, where a Proportion can be obtained, although it be only by way of Checque upon other Modes of working.

PROBLEM XXXIV.

The Latitude the Ship is in, the Sun's Declination, and the apparent Time given, to find the Altitude, and Azimuth.

EXAMPLE.

The Latitude 20° 11′ North, the Sun's Declination 23° 29′ South, at 24 Minutes paſt Nine in the Morning; What is the Sun's Altitude at that Inſtant, and his Azimuth?

This Example being properly conſtructed, a Triangle will be produced, having two Sides and an Angle included, given, to find the third Side, and the Angle at the Zenith; that is,

Sides { Z P = 69° 49′ = the Comp. Latitude.
P ⊙ = 113 29 = ⊙'s Diſt. from North Pole.
Angle Z P ⊙ = 39 00 = 2h. 36′ = Time from Noon.

The Altitude Z ⊙ may be better found by the Second Method in Problem XXIX. that is, by letting fall a Perpendicular Z Y from Z, at the Extremity of the leſs given Side, upon the greater Side P ⊙, which will form two Right-angled Triangles.

Then it will be,

As, S. Latitude ·· Radius : : S. Hour from 6 ·· T. 4th Arc.
S. 20° 11′ ·· S. 90° : : S. 51° ·· T. 64° 41′

Which fourth Arc P Y, thus found, and ſubtracted from P ⊙, becauſe the contained Angle is Acute, leaves the Reſidual Arc; then, as explained in Problem XXIX. it will be,

As, S.Co. 4th Arc=PY ·· S.Co. Reſidual Arc=Y ⊙ : : S.Co. leſs given Side=P Z ·· S.Co. Z ⊙ = the Altitude required.
S. 25° 19′ ·· S. 41° 12′ : : S. 20° 11′ ·· S. 32° 6′.

Theſe two Proportions are readily diſpatched by the Scale, when the Altitude will be found equal to 32° 06′, and this determined, the Angle at Z (the Azimuth) may be found by oppoſite Sides and Angles, that is,

As, S. Co. Altit. = Z ⊙ ·· S. Hour from Noon = Z P ⊙ : : S. ⊙'s Diſt. from elevated Pole = P ⊙ ·· S. ⊙'s Azim. from the S. = Suppl. of P Z ⊙.
S. 57° 54′ ·· S. 39° 00′ : : S. 180—113° 29′=66° 31 ·· S. 42° 57′ S. E.erly, becauſe the Angle at Z is obtuſe, and in the Morning.

PROBLEM XXXV.

The Sun's Declination, two Altitudes, and the Time between the two Obſervations given; the Latitude of the Place required.

EXAMPLE.

On a Day when the Sun's Declination was 20 Degrees North, in the Forenoon the Sun's Altitude was obſerved to be 18° 30′, and three Hours after, his Altitude was 44° 00′; What is the Latitude of the Place?

The Solution of this Problem depends upon three Triangles, in each of which ſomething is ſucceſſively to be found before we can come at the Latitude, and as it may be difficult to convey a clear Idea of their Connection, or Relation to each other, by Letters and Words, I ſhall therefore give the Conſtruction of the Figure at large.

CONSTRUCTION. *Pl.* I. *Fig.* 6.

Draw the primitive Circle with a Chord of 60 Deg. which muſt repreſent the Hour Circle on which the Sun was at the firſt Obſervation; quarter it with ÆQ the Equator, and PS the Axis, draw ⊙ *a* the Parallel of the Sun's Declination, which points out the Sun's Place, ⊙ 1 at firſt; then with the Secant of 45 Degrees, or 3 Hours; deſcribe the Oblique Circle P ⊙ 2 S, which will be the proper Hour Circle of the Sun, at the ſecond Obſervation, three Hours after the firſt; where this cuts the Parallel of Declination, at ⊙ 2, gives the Sun's Place then; about ⊙ 1, as a Pole, deſcribe a ſmall Circle parallel to a right Circle, at the Diſtance of 71° 30′, equal to the firſt Zenith Diſtance; and again, about ⊙ 2, as a Pole deſcribe another ſmall Circle, at the Diſtance of 46° 00′, equal to the ſecond Zenith Diſtance; the Point of Interſection of theſe two ſmall Circles at Z, gives the Zenith of the Place of Obſervation: Through Z ⊙ 1, Z ⊙ 2, ⊙ 1, ⊙ 2, and P Z, deſcribe great Circles, and P Z will be the Complement of the Latitude required.

Here then are three Triangles to work in, viz. ⊙ 1, ⊙ 2, P; ⊙ 1, ⊙ 2, Z; and ⊙ 2, P, Z.

Firſt, in the Iſoceles Spheric Triangle P, ⊙ 2, ⊙ 1.

$$\text{Given}\begin{cases}\text{Sides}\begin{cases}\text{P, ⊙ 1} = 70^\circ\ 00'\\ \text{P, ⊙ 2} = 70\ \ 00\end{cases}\\ \text{Angle ⊙1, P, ⊙2} = 45\ \ 00\end{cases}$$

Required Angle ⊙ 1, ⊙ 2, P and Side ⊙ 1, ⊙ 2.

Suppoſe a Perpendicular P 6, is drawn, which forms two Right-angled Triangles.

Then by Lord *Napier*, for the Angle P, ⊙ 2, 6.

As, T.Co. ⊙2, P 6 .. Rad. : : S.Co.P, ⊙2 .. T.Co.P, ⊙2, ⊙1.
T. 67° 30′ .. S.90° : : S. 20° 00′ .. T. 8° 04′
Subt. from 90 00

Rem. < P, ⊙ 2, ⊙ 1 = 81 56

For the Side ⊙ 2, 6 = $\frac{1}{2}$ the Side ⊙ 1, ⊙ 2.

As, Radius .. S. ⊙ 2, P : : S. ⊙ 2, P, 6 .. S. ⊙ 2, 6
S. 90° .. S. 70° 00′ : : S. 22° 30′ .. S. 21° 04′$\frac{1}{2}$, which

doubled, gives the Side ⊙ 1, ⊙ 2 = 42 09

Or, the Angle above may be determined by the Sines of oppoſite Sides and Angles in the Iſoceles Triangle, and then it will turn out to be 81° 57′, which Difference ariſes by not working by the Logarithms preciſely to Seconds in both.

Second,

In the Oblique Spheric Triangle Z, ⊙ 2, ⊙ 1.

$$\text{Sides given}\left\{\begin{array}{l} Z,\ \odot\,1 = 71^\circ\ 30' \\ Z,\ \odot\,2 = 46\quad 00 \\ \odot\,2,\ \odot\,1 = 42\quad 09 \end{array}\right.$$

Required the Angle Z, ⊙ 2, ⊙ 1.

Here are three Sides given, to find an Angle, to perform which, obſerve the Rules and Directions given in Prob. 23, 24, or 27, and 28, the latter of which are more immediately applicable to the Scale.

The preparative Work in order to find the half Sum and Remainder, being done as directed in Prob. 27.

The Proportions are,

As, Radius ·· S. Co. of one of the Sides containing the required Angle : : the S.Co. of the other containing Side ·· fourth Sine; and then,

As, this fourth Sine ·· S. ½ Sum of Sides : : S. Remainder ·· ſeventh Sine. In this Triangle more ſhortly expreſſed, thus,

Radius ·· Sine ⊙ 2, ⊙ 1 : : Sine Z, ⊙ 2 ·· 4th Sine; and as,
S. 90 ·· Sine 42° 9′ : : S. 46° 00′ ·· S. 28° 52′

Fourth Sine ·· S. ½ Sum Sides : : S. Rem. ·· ſeventh Sine.
S. 28° 52′ ·· S. 79° 49′ ½ : : S. 8° 19 ½ ·· S. 17° 10′, againſt which on the Line of Verſed Sines, will be found 114° 11′ the Angle Z, ⊙ 2, ⊙ 1, required.

Then, from this whole Angle ſubtract the Angle P, ⊙ 2, ⊙ 1, = 81° 56′ before found, the Remainder will be 32° 15′ = to the Angle P, ⊙ 2, Z.

Laſtly, in the Oblique Spheric Triangle P, Z, ⊙ 2, there are given two Sides and an Angle included.

$$\text{Given}\begin{cases}\text{Sides}\begin{cases}P,\ \odot\ 2 = 70^\circ\ 00' \\ Z,\ \odot\ 2 = 46\ \ 00\end{cases}\\ \text{Angle } P,\ \odot\ 2,\ Z = 32\ \ 15\end{cases}$$

The Comp. of the Latitude P Z, is required.

This Caſe is to be performed as explained in Problem 29; but by the Scale, obſerve the Directions given in the latter Part of the ſaid Problem, and the preparative Work being done, with a Perpendicular let fall from Z to P, ⊙ 2, the Proportions will be,

Firſt,

As, Radius ·· S.Co.P, ⊙ 2, Z : : T. Z, ⊙ 2 ·· Tan. 4th Arc.
T. 45 ·· S. 57° 45′ : : T. 46° 00 ·· Tan. 41° 13′, which ſubtracted from the greater Side P, ⊙ 2, gives the Reſidual Arc = 28° 47.

Second,

As, S. Co. 4th Arc ·· S. Co. Reſidual Arc : : S. Co. leſs given Side Z ⊙ 2 ·· S. Co. P, Z.
S. 48° 47′ ·· S. 61° 13′ : : S. 44° 00′ ·· 54° 02′ Latitude required, N.

This Caſe determines the Latitude truly, when the Ship is lying to, or at Anchor, but if under Sail, and has made any Diff. of Longitude or Latitude during the Interval of the two Obſervations, it muſt be taken Notice of; the whole of which then, to a Perſon not well verſed in Aſtronomy, may be found troubleſome and abſtruſe; therefore, to render this uſeful Problem eaſy and practicable, an ingenious *Hollander* was induced to compute (ſome few Years ago) a Set of *Logarithmic Solar Tables*, whence by a Method of Approximation, and taking the *Dead Reckoning* Latitude into the *Data*, the true Latitude may be readily found; which Pamphlets, containing Inſtructions, are ſold by Meſſrs MOUNT and PAGE, on Tower-Hill, or Meſſrs. NAIRNE and BLUNT, Cornhill.

In the Practice of Teaching, or for young Students, it is very eſſential to have the Diagram or Figure of what they are about, before them, as it conveys clearer Ideas, and aſſiſts the Judgment; Example is ſaid to be better than Precept, but if it is not ſo here altogether, yet it ſerves to confirm and eſtabliſh the latter; hence it may be wiſhed by ſome, that the Conſtructive Part had been treated of here, but had this been done, it muſt have extended through Plane and Spherics, which would have rendered this ſmall Treatiſe too

Voluminous, being chiefly intended for the Uſe of the Scale, yet this Defect (ſhould it be ſo found by any young Student) may be amply repaired by Application to ſundry Authors, who have wrote upon this Subject, (as heretofore noted in Page 14) and particularly to *Atkinſon's* Epitome, *Hodgſon's* Syſtem of the Mathematics, and my late worthy Friend, *Robertſon's* excellent Elements of Navigation. who are very copious herein, as well as in the Principles of Plane and Spheric Trigonometry, with which the Tyro ought to be acquainted before he proceeds to the further Study of any Branch of Mathematical Science; to theſe and others, then, I recommend him, and beg Leave to draw towards a Concluſion, having rather exceeded the Limits of the original Deſign, but not without ſome Hopes that every Difficulty relative to Inſtrumental Operations, will, by theſe Papers, be totally removed.

THE

USE OF THE SCALE IN DIALLING.

THE Uſe and Application of three Lines remain yet to be explained.—Theſe are, the Lines of Latitudes, Hours, and Inclination of Meridians, which were mentioned in the Deſcription, Page 5, as peculiar to DIALLING; the Geometric Conſtruction of which may be ſeen in Mr. *Robertſon's* Book on the Uſe of Mathematical Inſtruments, Page 28.

To enlarge upon Inſtrumental Dialling, would require ſeveral other Lines to ſupply the proper Requiſites, which could not conveniently be put upon *this* Scale, without diſcharging others more eſſential to the preſent Plan of Buſineſs; and as there are Dialling Scales profeſſedly made for that Purpoſe, ſuch as *Collins's*, &c. I muſt beg Leave to refer thoſe Readers thereunto, who may be deſirous of further Knowledge in this kind of *Gnomonics*.

Hence, then it may with ſome apparent Reaſon, be remarked, "that *theſe* Lines are altogether ſuperfluous, if they cannot here be made Uſe of."

To ſatisfy this Remark, it is to be obſerved, that although theſe three Lines alone are not ſufficient for the Conſtruction of *Decliners*, *Incliners*, and *Recliners*, yet the two firſt, *viz.* Hours, and Line of Latitudes (being general for all Dials with Centers) may be readily and eaſily applied to the Conſtruction of *Horizontals*, and *North* or *South Erect Direct*, without the Help of any others; and as ſome curious Mariner, or other Reader, may be deſirous of an Explanation, he ſhall be obliged with ſufficient Directions under another Problem or two for this Purpoſe, which will finiſh the Deſcription, and (in ſome Meaſure the) Uſe of *this excellent Practical Scale*, and conſequently of this Treatiſe.

PROBLEM XXXVI.

To conſtruct an Horizontal Dial for any given Latitude.

EXAMPLE.

An Horizontal Dial for *London*, in Lat. 51° 32′ North, is required to be made?

DIRECTIONS. *Plate* IV.

Draw C D for the Hour Line of Six, and interſect it at Right-angles with A B, the Meridian or Hour Line of Twelve; then with your Compaſſes, take 51° 32′ from the Scale or Line of Latitudes, equal to the Stile's Height, and lay this off both ways from the Center B, to C and D: Then take the whole Scale of ſix Hours, and ſet that Diſtance from C to A, and from D to A, and draw the Lines C A and D A; upon theſe two Lines, from the Points C and D, lay off the ſeveral Hour-Diviſions, taken from the Hour-Line, and mark the Points thus ſet off, with the Figures, 1, 2, 3, 4, 5, and 11, 10, 9, 8, 7, &c. then Lines drawn from the Center B, through theſe Diviſions, will be the true Hour Lines: In like Manner you may lay off for Halves and Quarters, &c.

Note 1. That the Height of the Stile or Axis (in all Horizontal Dials) is always equal to the Latitude of the Place, and the Sub-ſtile (upon which the Gnomon or Stile ſtands) is the Meridian or Hour Line of 12; therefore to conſtruct the

Stile, deſcribe a Semi-circle or Quadrant, with a Chord of 60 Degrees; then take in your Compaſſes the Latitude (51° 32′) from the ſame Line of Chords, and lay it thereon from F to S, that is, from the Meridian, or 12 Line, to near 4, and draw the Line B S, which gives the Height of the Gnomon, and may be made of Tin, Braſs, or Copper-plate, &c. See *Plate* IV.

Note 2. This Stile, or Hypothenuſe-Line of the Gnomon, muſt always lie parallel to the Axis of the World, hence its Extremity will point at the elevated Pole.

The ſame computed Trigonometrically.

The Hour-Diſtances from the Meridian on the Plane, may readily be computed by Logarithms or Gunter's Scale, the Operations being performed by the following Proportion, viz.

As, Radius .. S. Lat. of Place : : T. each Hrs. equat. Diſt. .. T. Hrs. Diſt. on Plane.

S. 90° .. S. 51° 32′ : :	T. 15° 0′ ..	T. 11° 51′
	30 0	24 19
	45 0	38 04
	60 0	53 36
	75 0	71 06
	90 0	90 00

Which Reſults or Hour Diſtances being taken from a Scale of Chords, and ſet upon the Plane (deſcribed or drawn with a Chord of 60 Deg.) from the Meridian, both ways from F, will give the Points 1, 2, 3, &c. 11, 10, 9, &c. as before; for between F and 11 or 1, you will find the Meaſure of 11° 51′ on the Chords, and from F to 10 and 2, the Meaſure of 24° 19′, &c.

The Halves and Quarters may be found in like Manner, eſtimating in the Ratio of 15 Degrees on the Equinoctial, to one Hour of Time.

The Gnomon or Stile formed as before, finiſhes your Dial; and in like Manner proceed for any other Latitude.

If the Reader underſtands the Stercographic Projection of the Sphere, the former Methods may be chequed or proved by ſuch Projection upon the Plane of the Horizon, and for the given Latitude (in this Example, *London.*) For the Horizon being drawn, (which is the Dial Plane) and quartered with the Prime Vertical and Meridian, and on this laſt, the Complement of the Latitude laid off from the Center (Zenith) gives the Place or Poſition of the elevated Pole; at 90 Degrees from which, draw the Equinoctial, (an oblique Circle;) then draw Meridians (oblique Circles) through the Pole, making Angles thereat, and with each other, of 15 Degrees, or one Hour of Time, (and for Halves and Quarters in like Manner, if neceſſary.) Theſe Meridians thus drawn, will interſect the Horizon, and therein point out the Place of the Hours, &c. through which Points or Interſections, ſtreight Lines (Azimuth Circles) being drawn from the Center (Zenith) and continued through a circular, or rectangular circumſcribed Space (for the Hour Figures) will be the proper Hour Lines.

The Axis or Stile drawn upon this Plane, as before directed, will compleat the Dial, and confirm or detect the two former Methods of Conſtruction.

PROBLEM XXXVII.

To conſtruct a South or North Erect Direct Dial for any Latitude.

EXAMPLE.

A South Erect Direct Dial for *London*, in Lat. 51° 32′ N. is required to be made?

The Situation or Poſition of theſe Dials is againſt an upright Wall, whoſe Aſpect is due North or South, and they are to be conſtructed or computed exactly as explained in the laſt Problem for Horizontals, excepted only, that in *theſe* the Complement of the Latitude muſt be applied or made Uſe of, inſtead of the Latitude, as in the former; hence, the Height, Elevation, or Diſtance of the Stile from the Meridian, in this Example, is 38° 28′; and it is further to be duly obſerved, that in all South Erect Direct, and alſo in all South Erect Decliners, the Stile or Gnomon muſt be placed on the South Side, and point downwards towards the South Pole; but in North, then on the North Side, and point upwards towards the North Pole.

If a North Plane be required, it may eaſily be conceived that *this* will be the South inverted, for if the South Stile (a Rod or Wire) was run through the Center in the oblique Direction it poſſeſſes, it will be the proper Gnomon on the North Side, and will ſtand above the Horizontal, or Hour Line of Six, and point upwards, making an Angle with the Meridian equal to the Complement of the Latitude; the Diſtances of the Hour Lines will be the ſame, and conſequently may be computed or found, and ſet off from the Six o'Clock Line, as before.

It is hoped that theſe Directions will be ſufficient without the Help of another Diagram.

FINIS.

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